ViT and Self-Supervised Models

CS280

Spring 2025

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Attention is expensive

For Memory and Computation.

Prevents long context!

Flash Attention

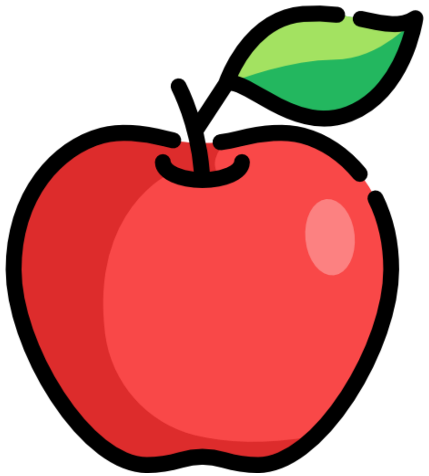
- Does not instantiate the entire N x N attention matrix A , i.e. QK^T
- Dedicated GPU kernel for efficient implementation
- Running sum for softmax…
- Memory reduced from O(N^2) to O(Nd)
- Need operation to be on Q and K independently

Dao et al. NeurIPS 2022…

Positional Encoding

I bought an apple watch

watch an apple I bought



Slide Credit: Jiabin Huang

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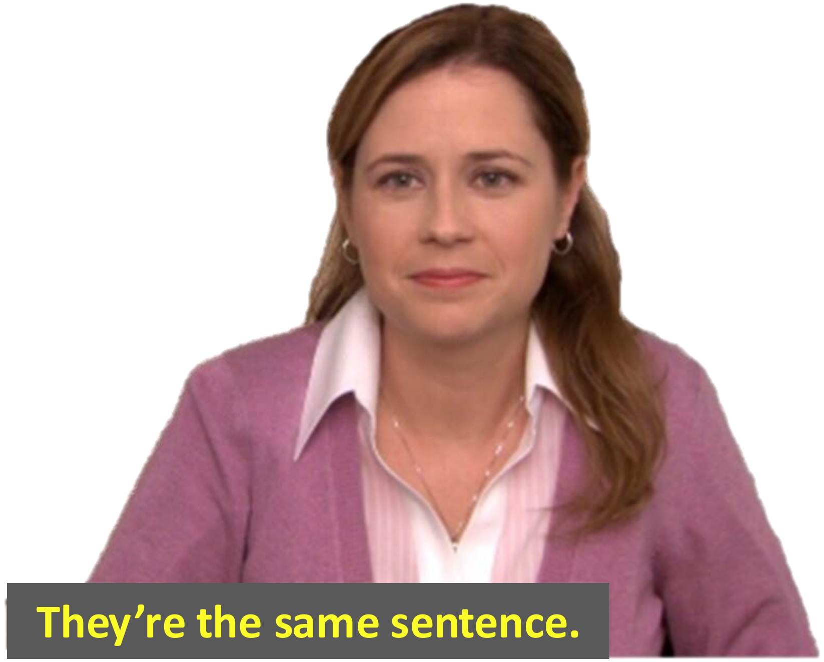


Slide Credit: Jiabin Huang

Embedded Tokens	$\boldsymbol{x}_1$	$\boldsymbol{x}_2$	$\boldsymbol{x}_3$	$\boldsymbol{x}_4$	$\boldsymbol{x}_5$
Tokens	**I**	**bought**	**an**	**apple**	**watch**
Position	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$

Position k

Angular frequency

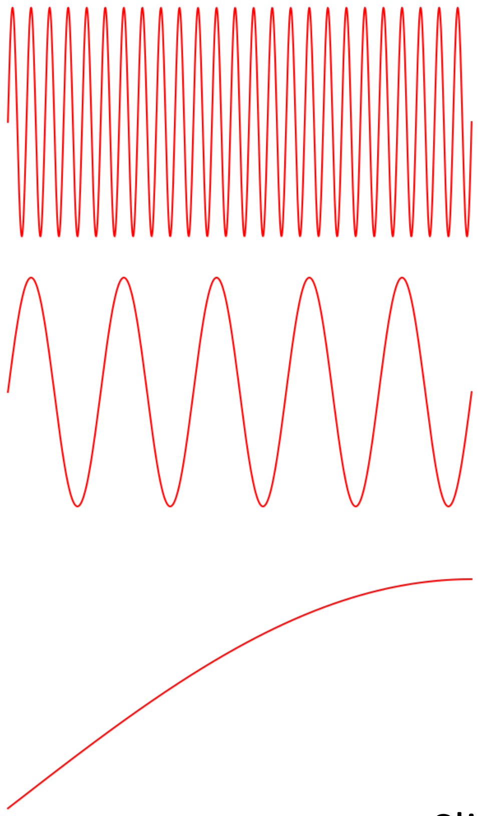
$$w_i = N^{-2i/d}$$

$$N = 100{,}000$$

$$d \quad \begin{bmatrix} \sin(w_0 m) \\ \cos(w_0 m) \\ \sin(w_1 m) \\ \cos(w_1 m) \\ \vdots \\ \vdots \\ \sin\left(w_{\frac{d}{2}-1} m\right) \\ \cos\left(w_{\frac{d}{2}-1} m\right) \end{bmatrix}$$

← Fast oscillating

← Slow oscillating



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Position m

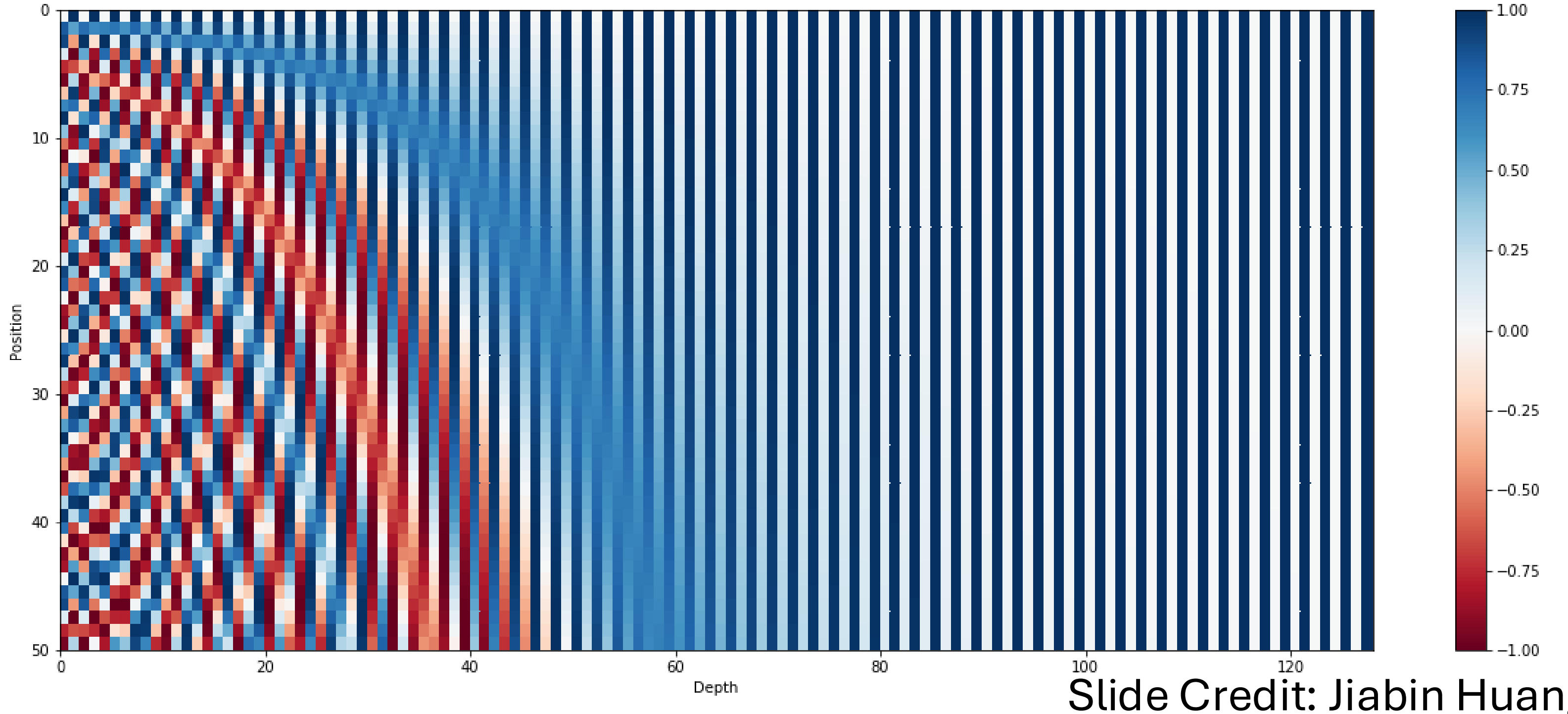
Angular frequency

$$w_i = N^{-2i/d}$$

$$N = 100{,}000$$

N is wavelength

$$d \left\{ \begin{bmatrix} \sin(w_0 m) \\ \cos(w_0 m) \\ \sin(w_1 m) \\ \cos(w_1 m) \\ \vdots \\ \vdots \\ \sin\left(w_{\frac{d}{2}-1} m\right) \\ \cos\left(w_{\frac{d}{2}-1} m\right) \end{bmatrix} \right.$$



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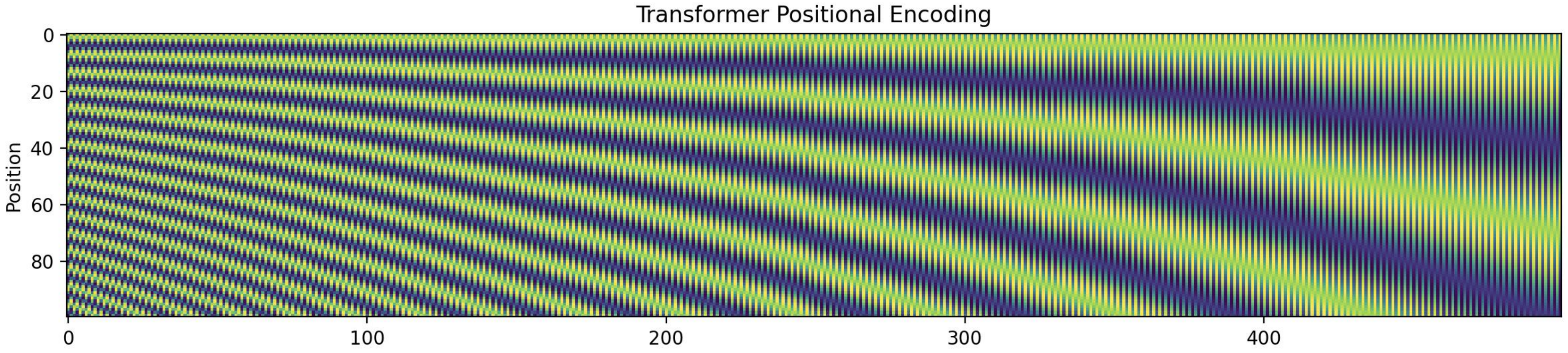
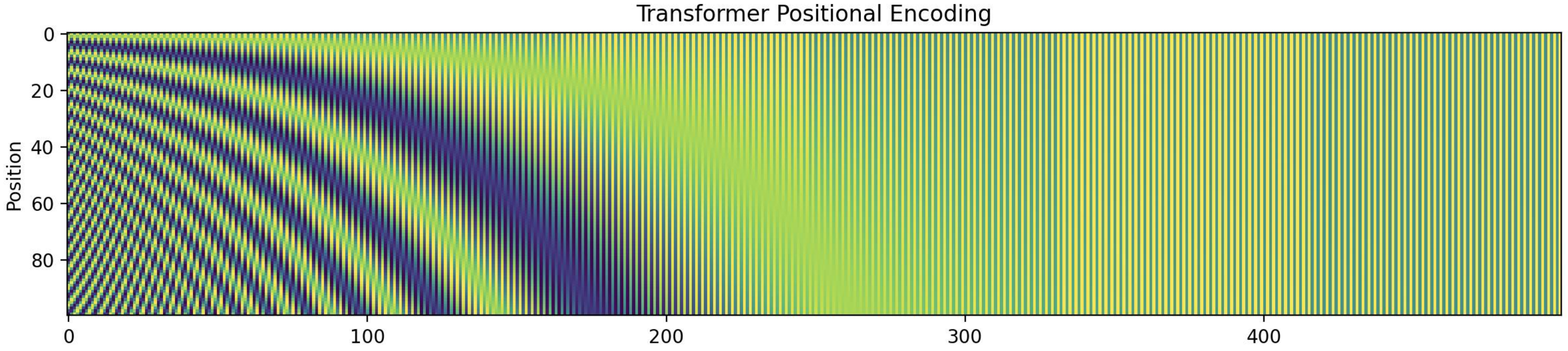
Changing N

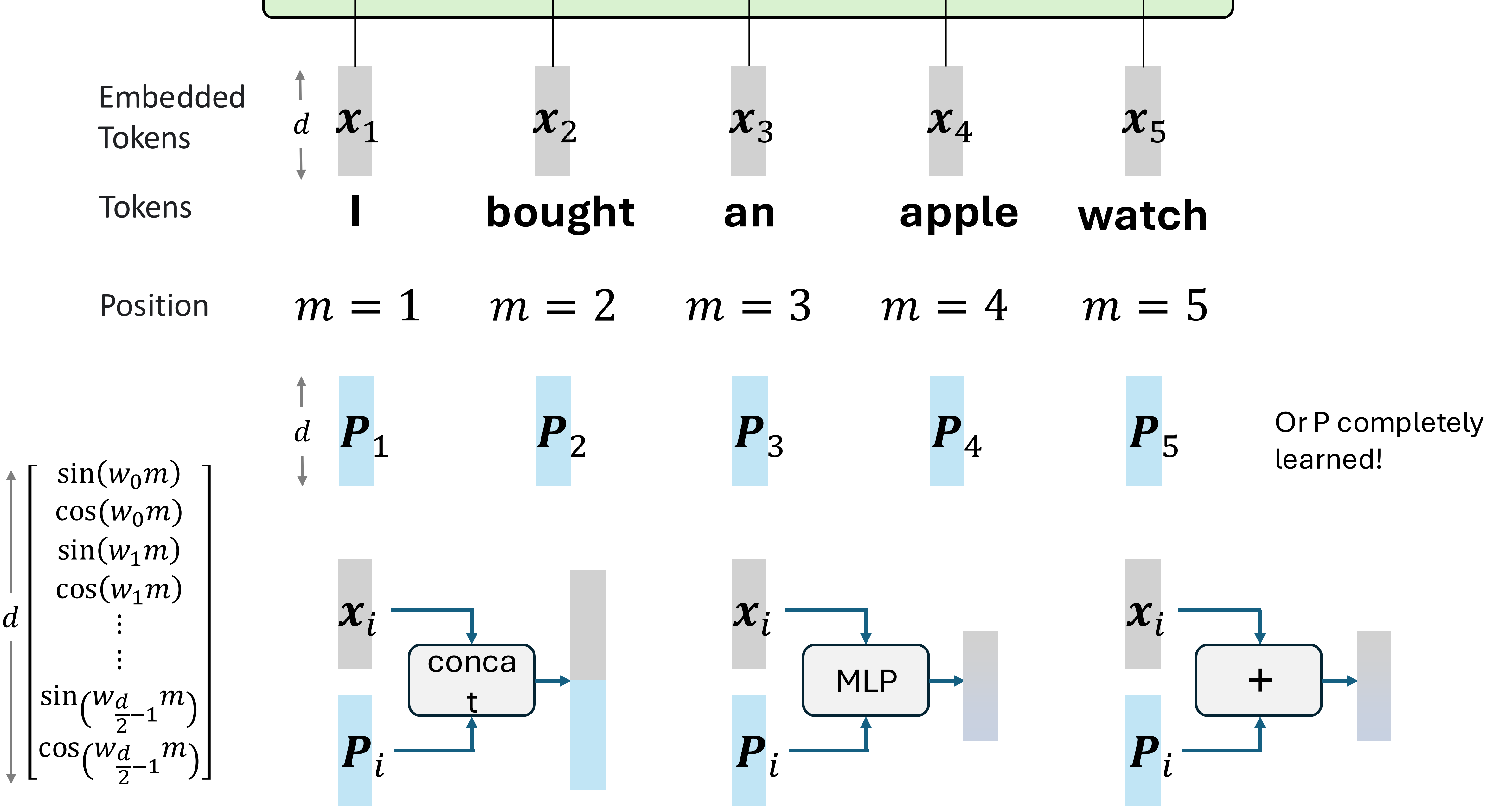
Goal: Unique positional encoding for each of your tokens.
N is the wavelength of the low-frequency band, larger it is, larger number you can represent.

N=10



What is used
N=10,000





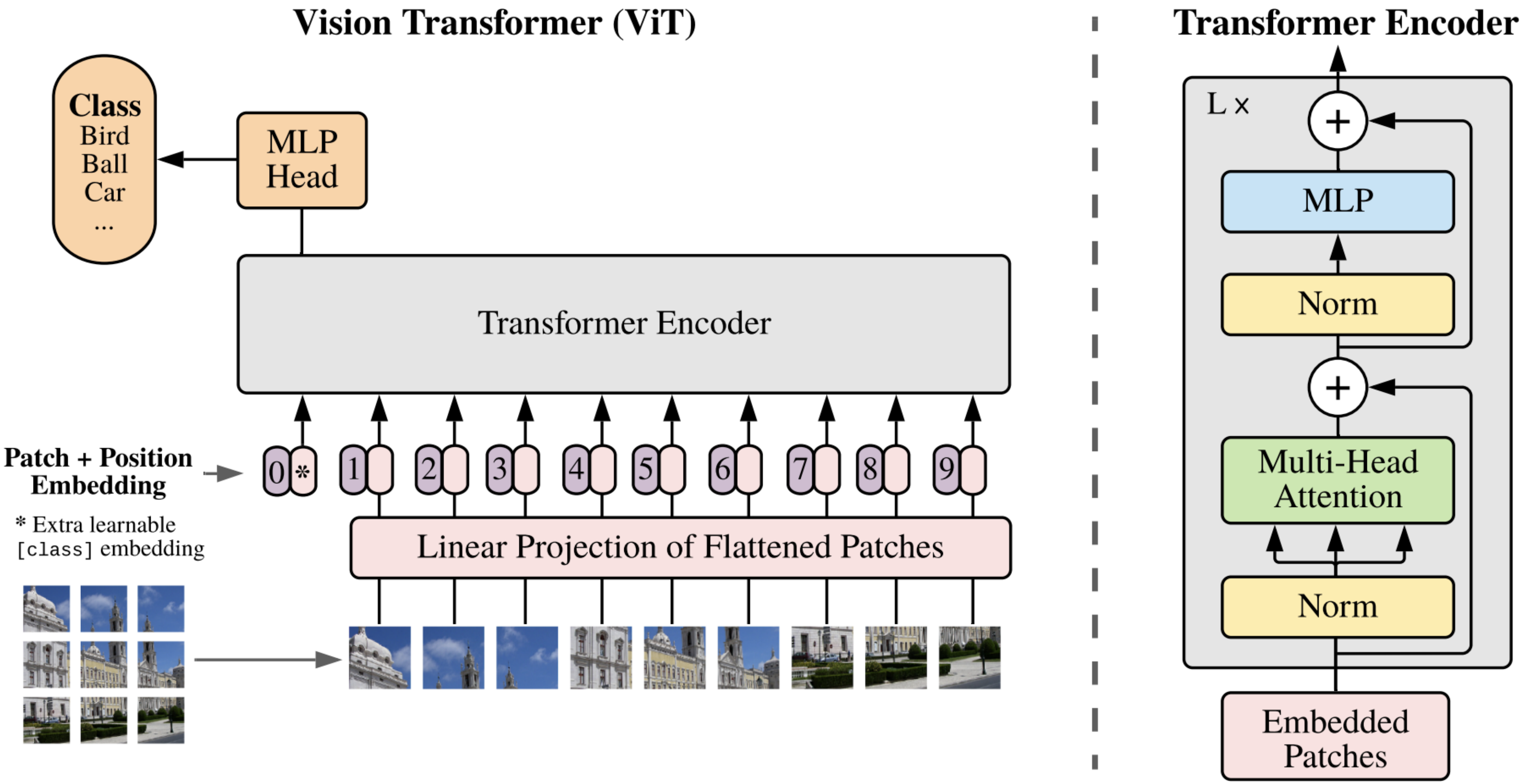
Slide Credit: Jiabin Huang

Back to pixels!

How to apply transformers to pixels?

- Naïve tokenization of a pixel for 224x224 image:
  - Over 500k tokens!
- Too much!
- Solution?

Vision Transformer



An image is worth 16x16 words, Dosovitskiy e tal. ICLR 2021

Positional Encoding

- Originally, they just flattened the patches from 1-256, treated it as a 1D signal!
- Or learned positional embedding

- Problem?
- Both solutions assume fixed size!
- You can’t change your image size!
- Need to retrain..

Relative POsitional Encoding: ROPE

Absolute Positional Encoding

Position	1	2	3	4	5	6
	I	**walk**	**my**	**dog**	**every**	**day**

Slide thanks to Jiabin Huang

Absolute Positional Encoding

Position	1	2	3	4	5	6
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	every	**day**	**I**	**walk**	**my**	**dog**

Slide thanks to Jiabin Huang

RELATIVe Positional Encoding

Position

I wal my dog every day
k

Position

every day I wal my dog
k

Slide thanks to Jiabin Huang

Re-write the original posenc

$$\mathrm{PE}(p) = \begin{bmatrix} \sin(w_0 p) \\ \cos(w_0 p) \\ \sin(w_1 p) \\ \cos(w_1 p) \\ \vdots \\ \sin\left(w_{\frac{d}{2}-1} p\right) \\ \cos\left(w_{\frac{d}{2}-1} p\right) \end{bmatrix} \Bigg\updownarrow d$$

- Focus on one frequency, $\theta = w_d$
- Then this is a 2D vector

$$\mathrm{PE}(p) = [\sin(\theta p), \cos(\theta p)].$$

- This is a coordinate on an unit circle!
- You can write this fourier basis in its exponential form (Euler's formula)

$$e^{i\theta p} = \cos(\theta p) + i \sin(\theta p).$$

- Has a nice property (holds across frequencies)!:

$$\langle \mathrm{PE}(p_1), \mathrm{PE}(p_2) \rangle = e^{-i\theta p_1} e^{i\theta p_2} = e^{i\theta(p_2 - p_1)}$$

Multiplying by $e^{i\theta}$ is same as applying a 2D rotation matrix

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ROPE — rotate tokens for relative positional encoding

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right)V.$$

$$Q = W_q X, \quad K = W_k X.$$

Absolute PosEnc:

$$X = \cancel{X' +} PE(m)$$

$$QK^T = (W_q X)(X^T W_k^T).$$

$$Q = W_q(Xe^{im}), \quad K = W_k(Xe^{in}).$$

$$QK^T = W_q X e^{i(m-n)} X^T W_k^T.$$

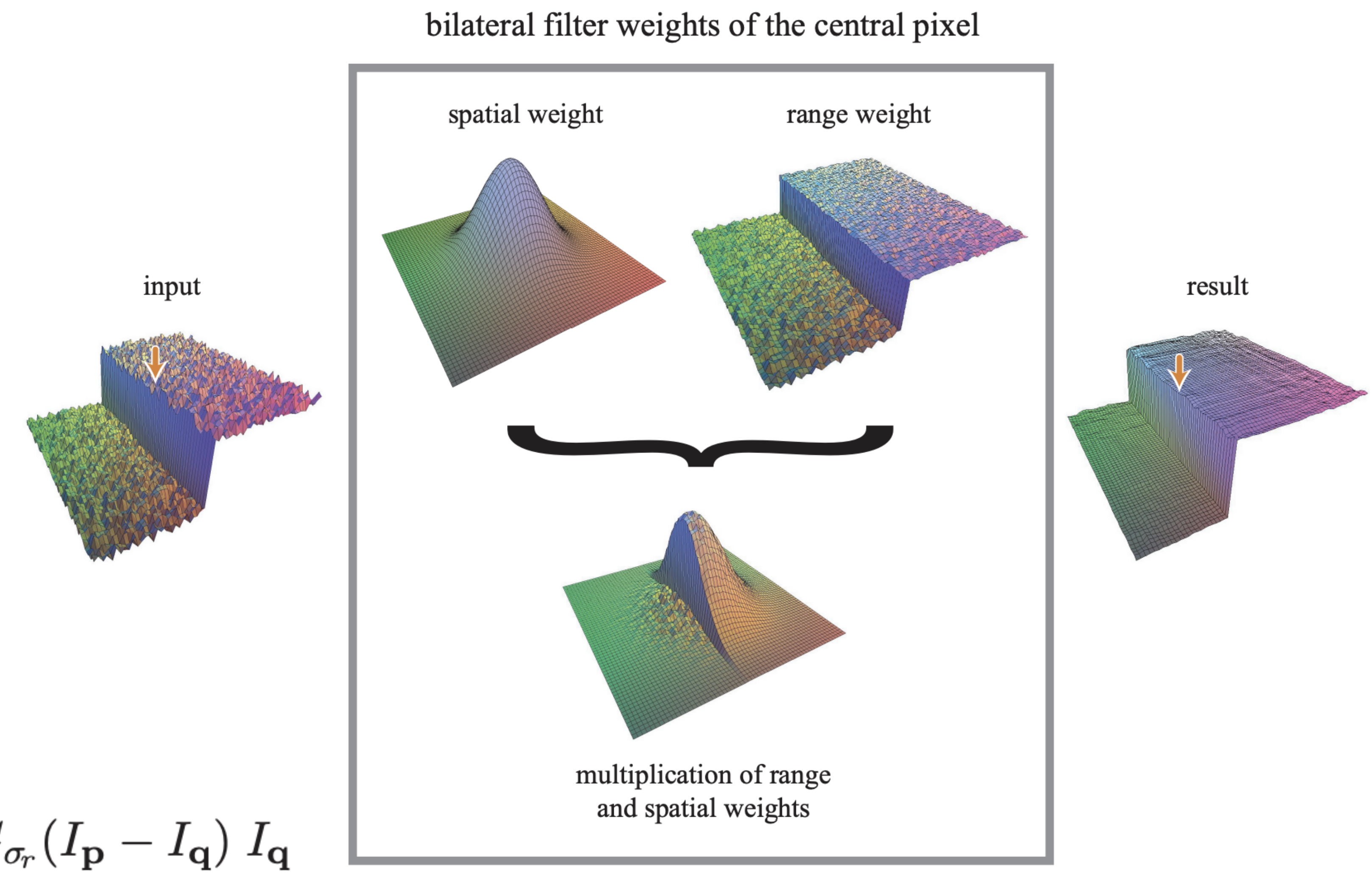
Pre-multiply the tokens with rotations

Flash Attention friendly!!

Unfortunately only works in 1D SO(2) bc of commutativitiy

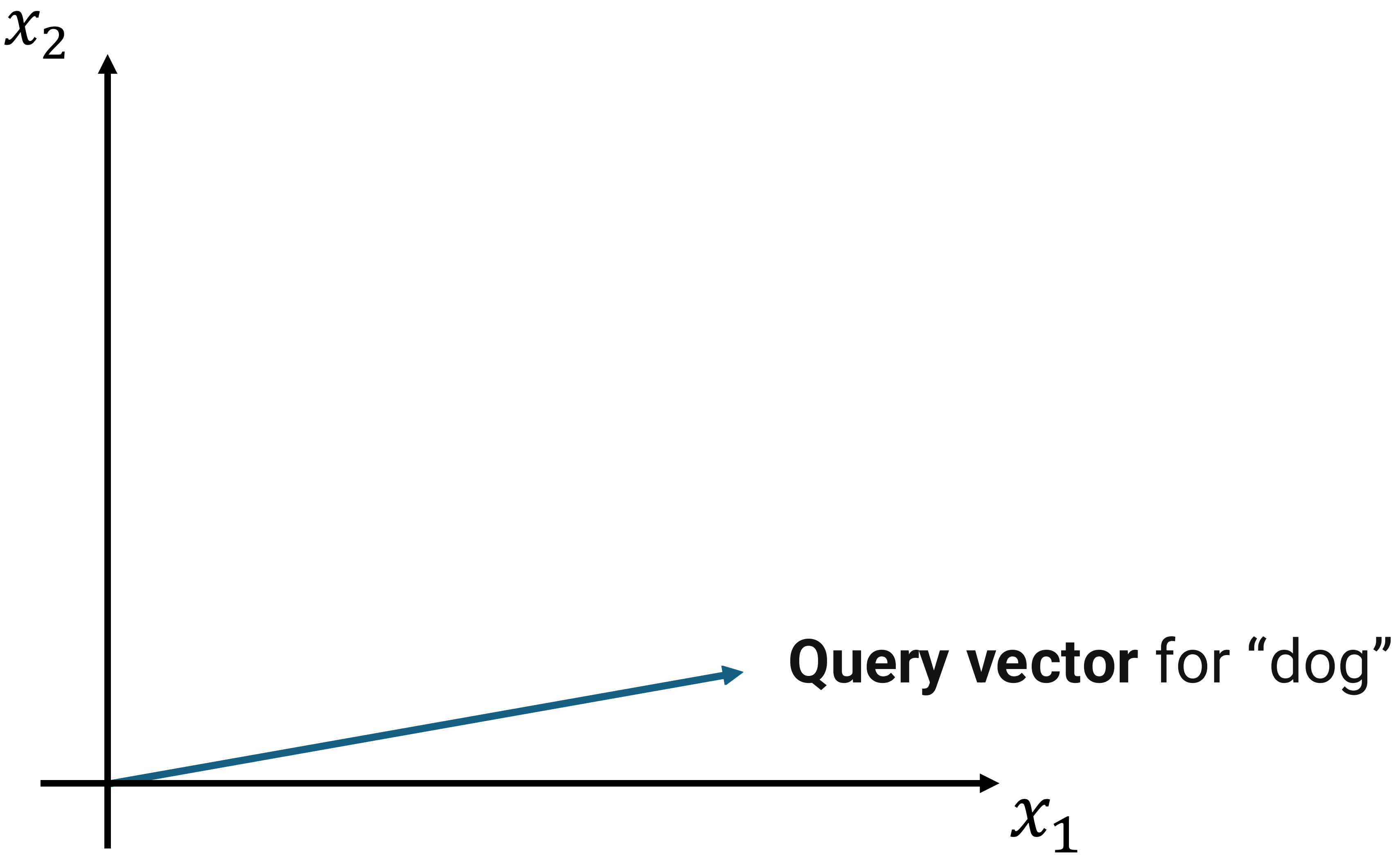
Aurich and Weule 1995
Tomasi and Manduchi 1998...

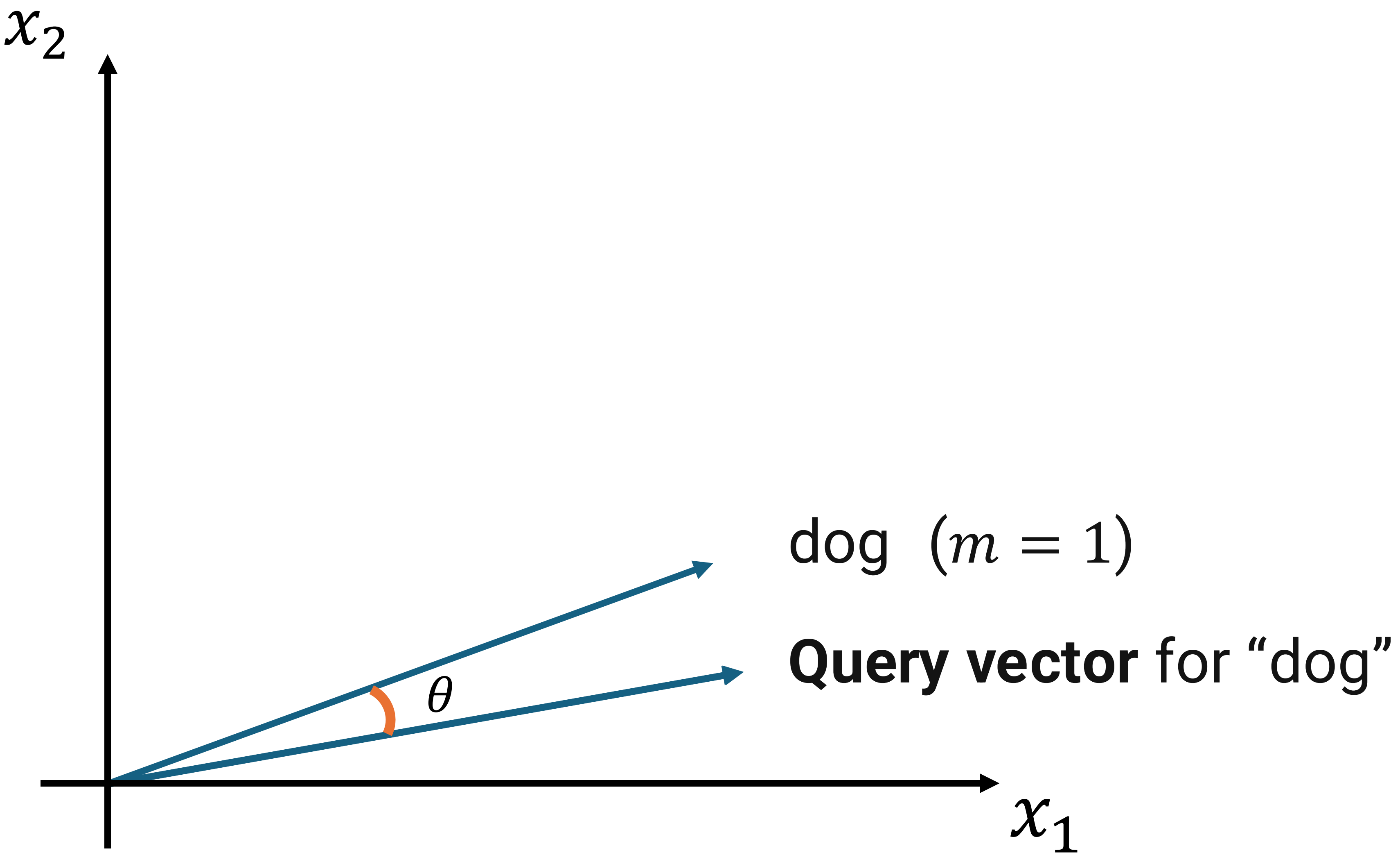
Bilateral Filter



$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in \mathcal{S}} \overbrace{G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|)}^{\text{Distance Similarity}} \, \underbrace{G_{\sigma_r}(I_{\mathbf{p}} - I_{\mathbf{q}})}_{\text{Pixel Similarity}} \, I_{\mathbf{q}}$$

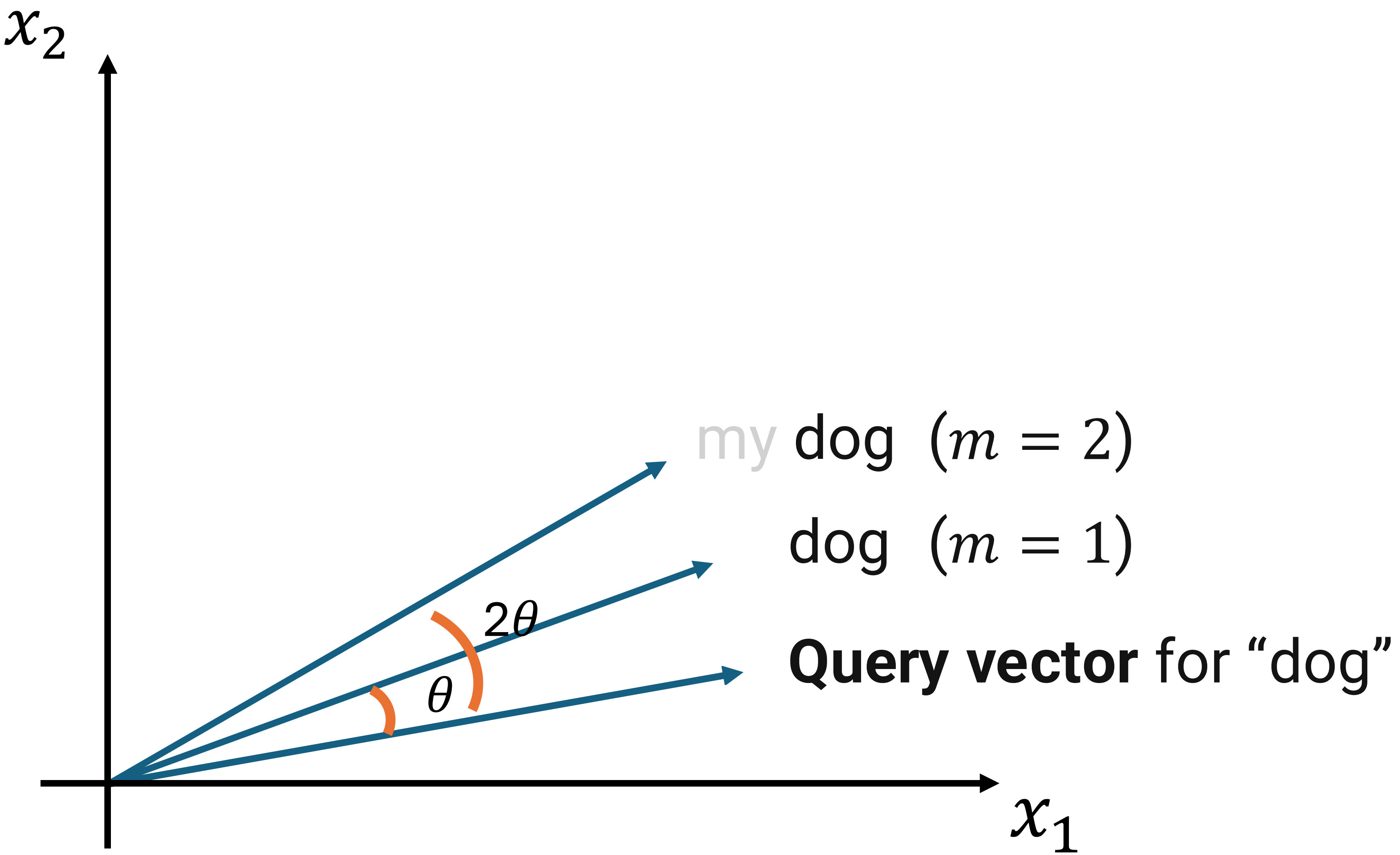
Figure from paris et al.

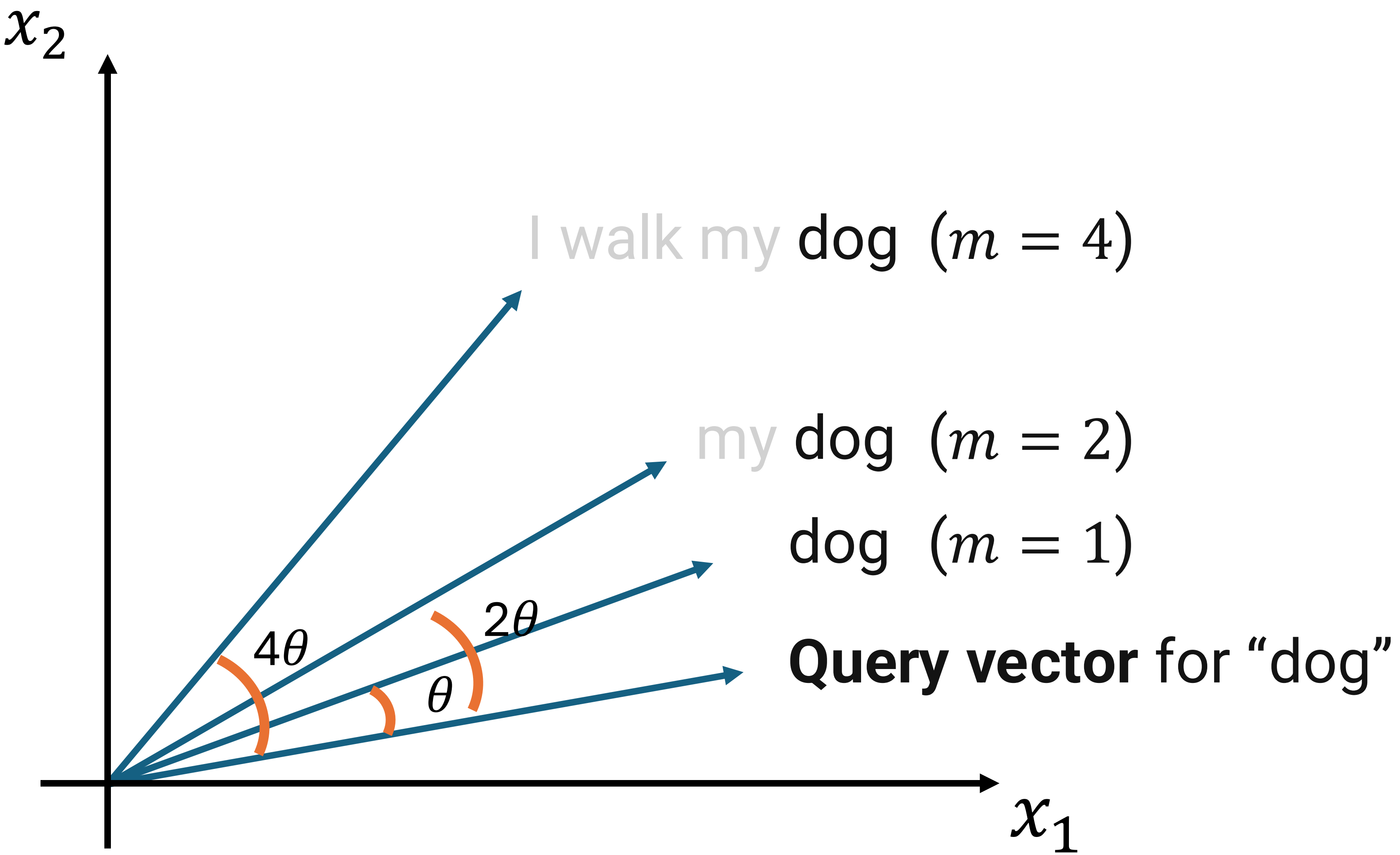

x_2
Query vector for “dog”
x_1



Slide thanks to Jiabin Huang

Rotary Positional Embeddings, 2021


x_2
my dog $(m = 2)$
dog $(m = 1)$
2θ
θ
Query vector for "dog"
x_1



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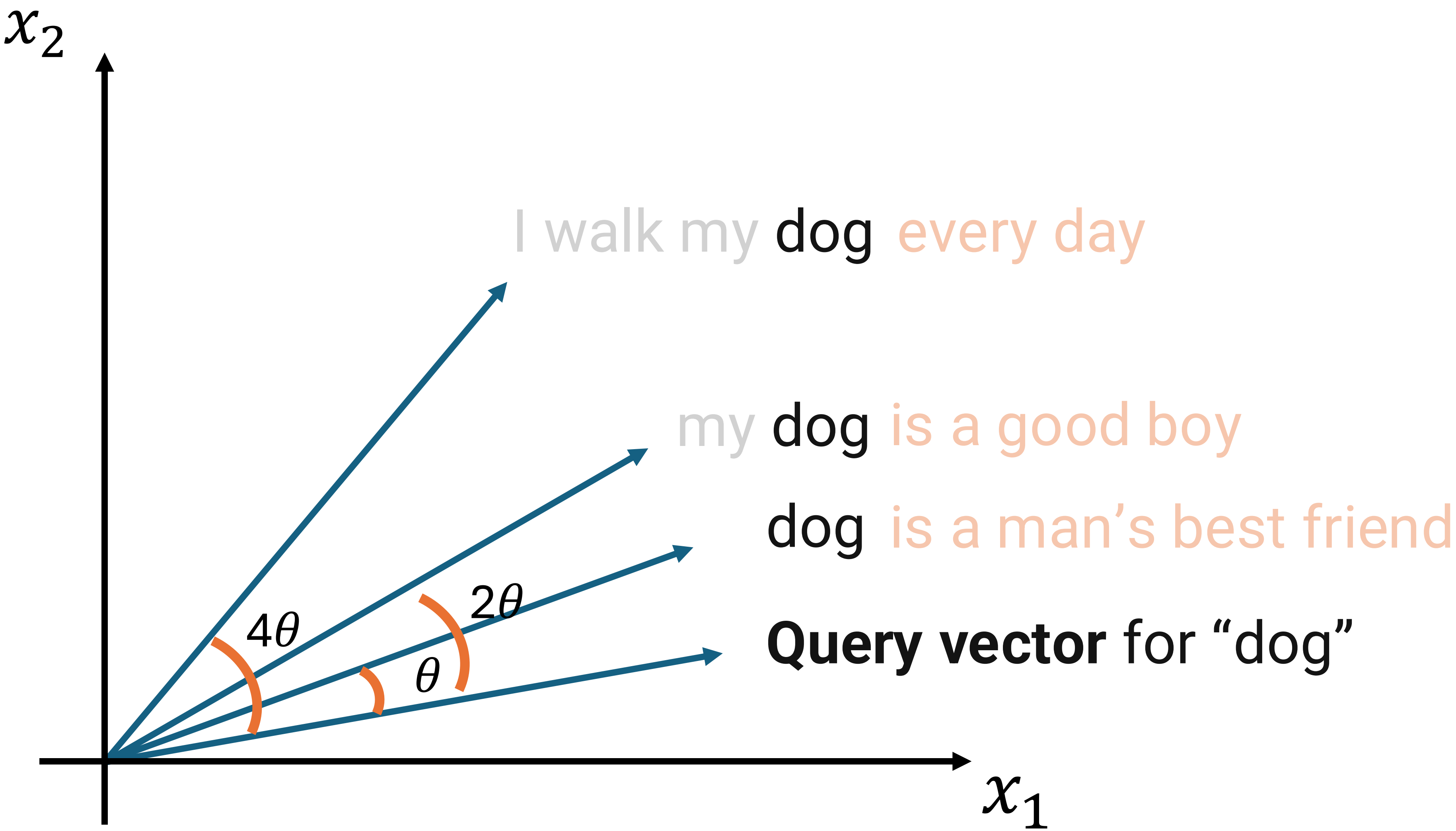

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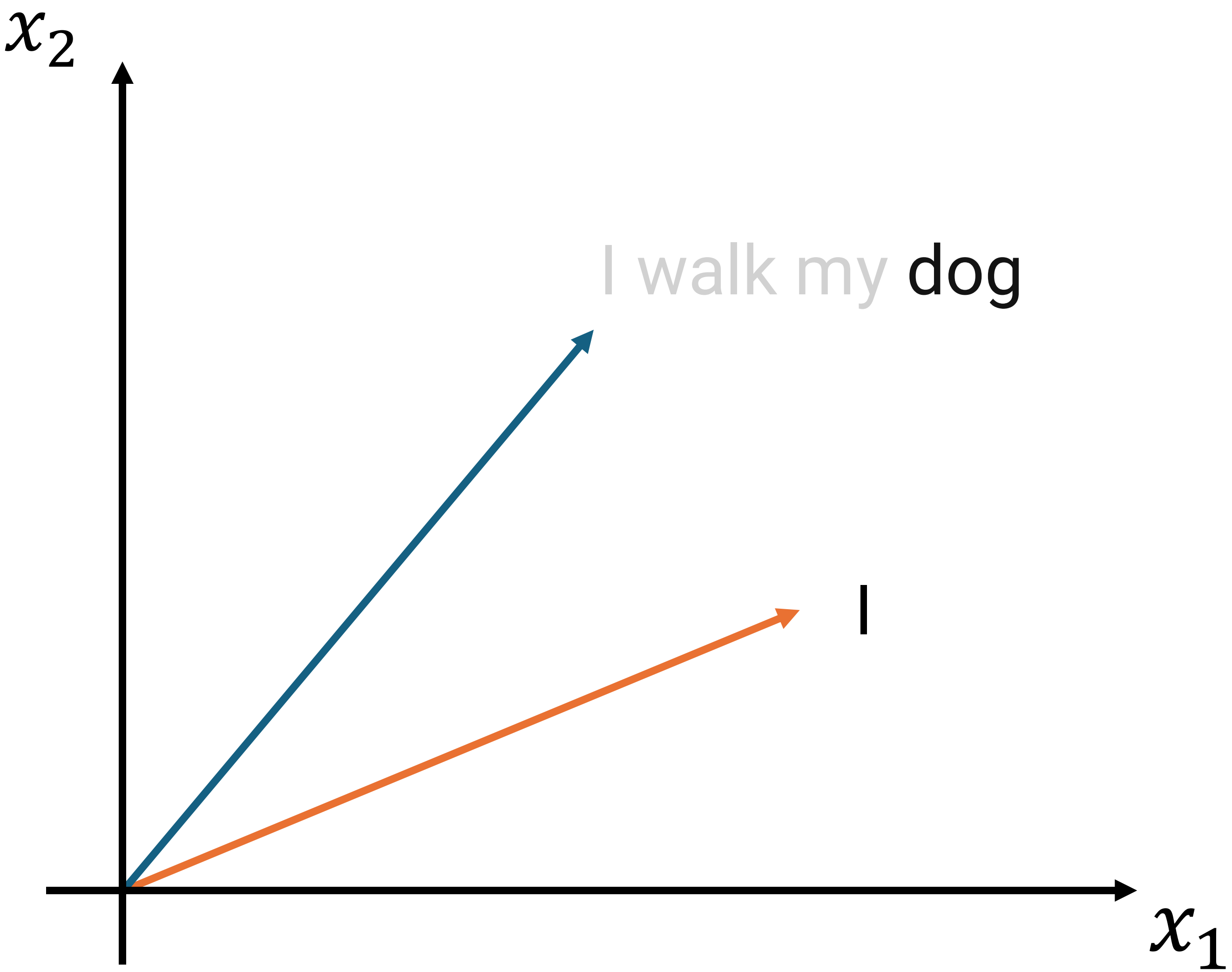

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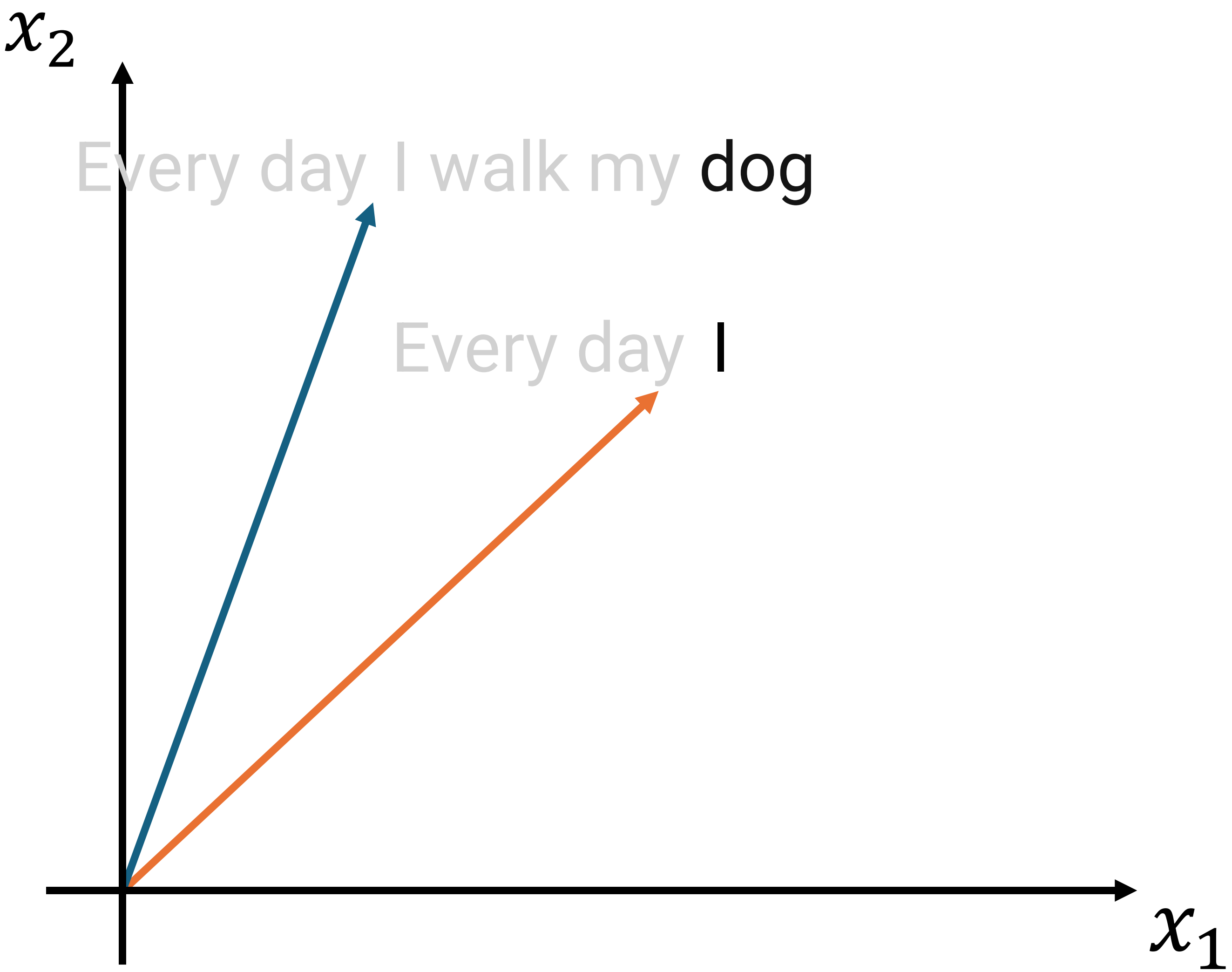


Slide thanks to Jiabin Huang

Rotary Positional Embeddings, 2021

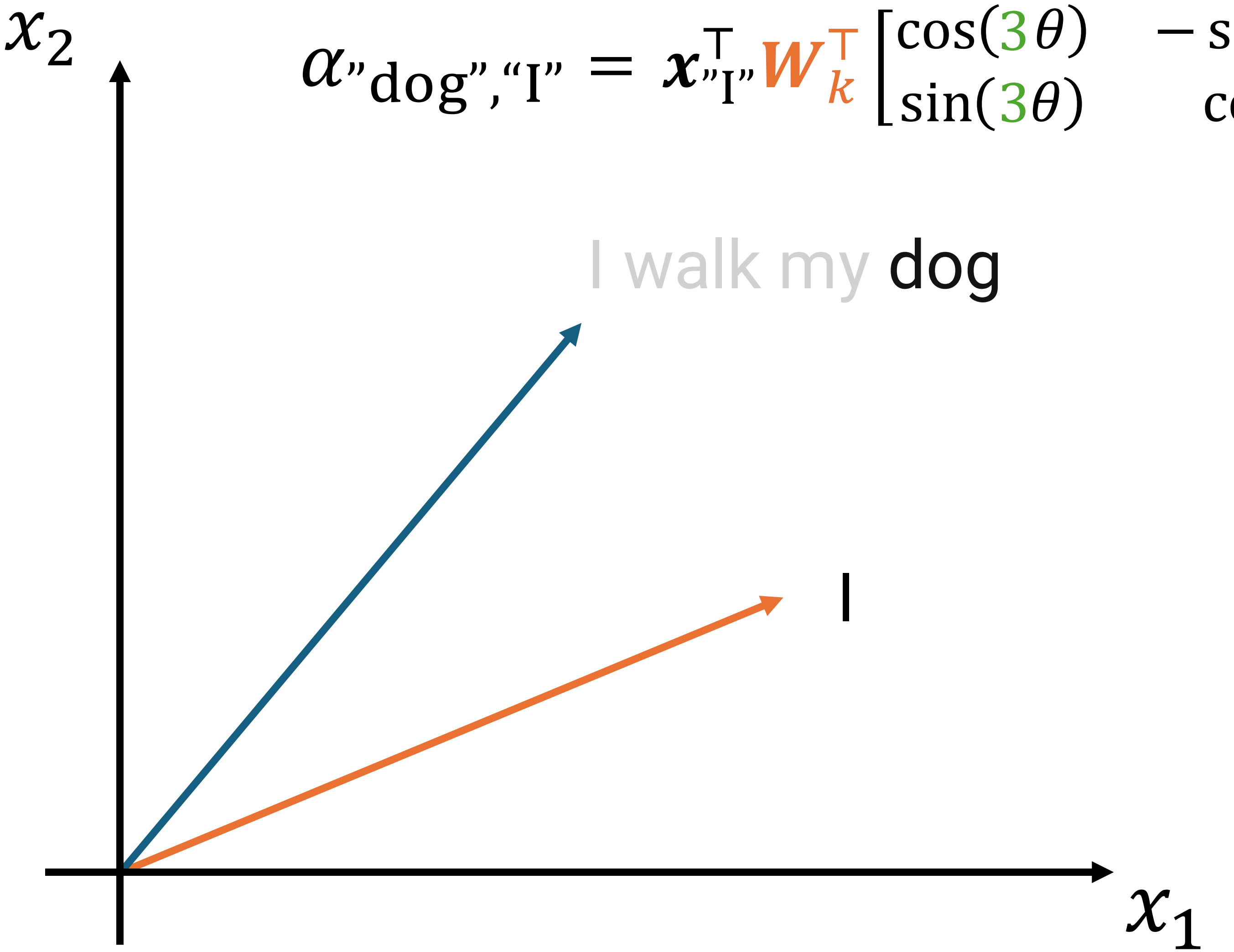
$$q_{\text{"dog"}} = \begin{bmatrix} \cos(4\theta) & -\sin(4\theta) \\ \sin(4\theta) & \cos(4\theta) \end{bmatrix} \boldsymbol{W}_q \boldsymbol{x}_{\text{"dog"}}$$

Query vector for “dog”

$$k_{\text{"I"}} = \begin{bmatrix} \cos(1\theta) & -\sin(1\theta) \\ \sin(1\theta) & \cos(1\theta) \end{bmatrix} \boldsymbol{W}_k \boldsymbol{x}_{\text{"I"}}$$

Key vector for “I”

$$\alpha_{\text{"dog","I"}} = \boldsymbol{x}_{\text{"I"}}^{\top} \boldsymbol{W}_k^{\top} \begin{bmatrix} \cos(3\theta) & -\sin(3\theta) \\ \sin(3\theta) & \cos(3\theta) \end{bmatrix} \boldsymbol{W}_q \boldsymbol{x}_{\text{"dog"}}$$



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$$q_{\text{"dog"}} = \begin{bmatrix}\cos(6\theta) & -\sin(6\theta)\\ \sin(6\theta) & \cos(6\theta)\end{bmatrix} \boldsymbol{W}_q \boldsymbol{x}_{\text{"dog"}}$$

Query vector for “dog”

$$k_{\text{"I"}} = \begin{bmatrix}\cos(3\theta) & -\sin(3\theta)\\ \sin(3\theta) & \cos(3\theta)\end{bmatrix} \boldsymbol{W}_k \boldsymbol{x}_{\text{"I"}}$$

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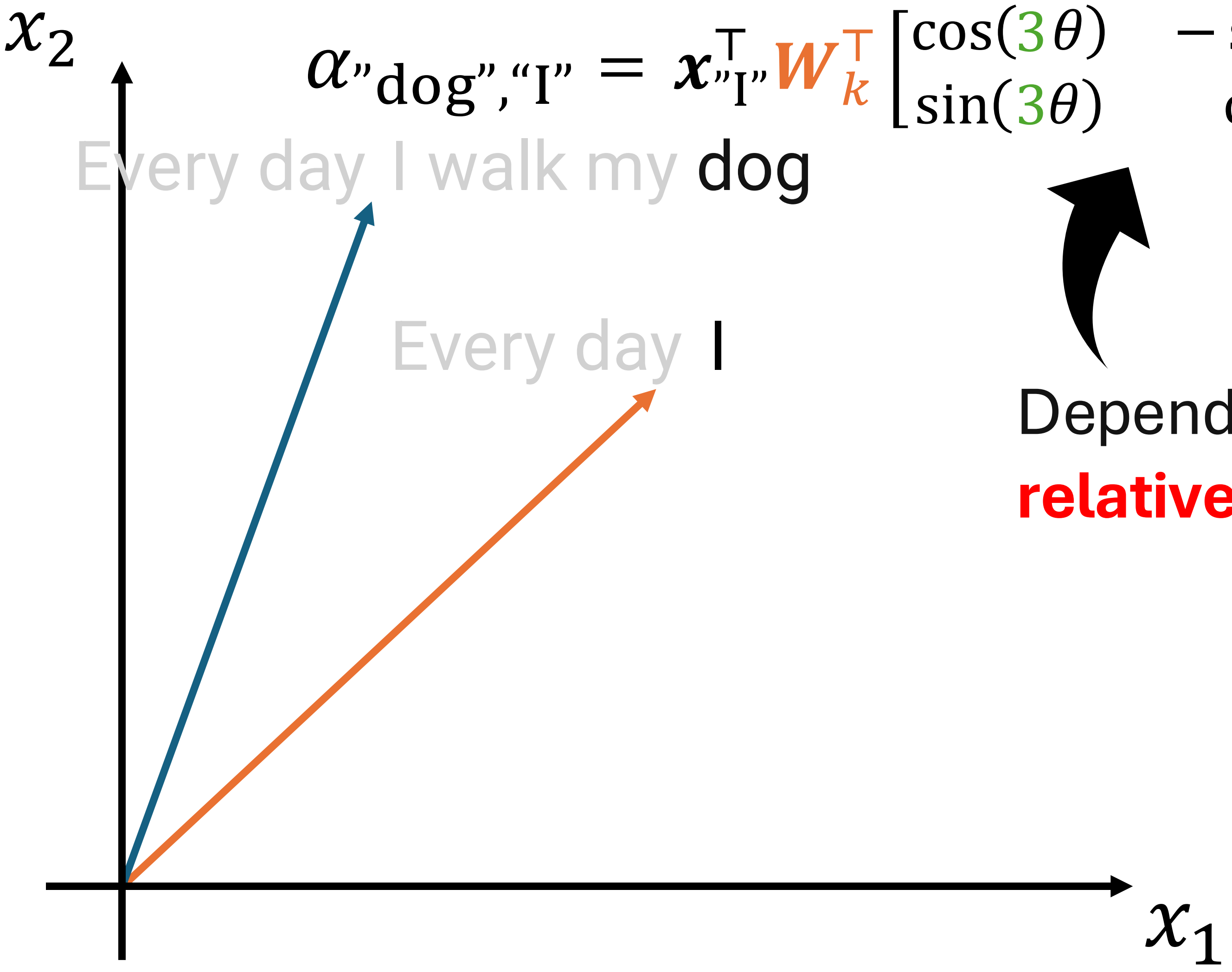


Depends only on the **relative positions**

Slide thanks to Jiabin Huang

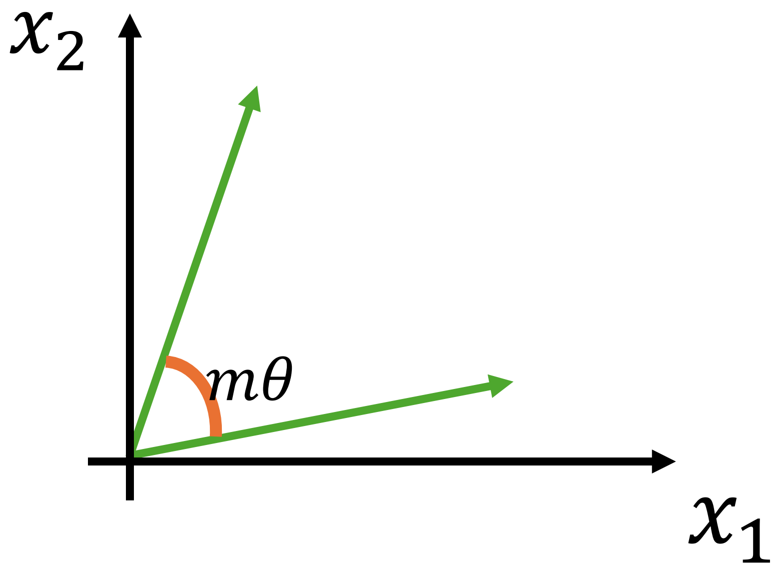
$$R_{\Theta}^{m} \boldsymbol{W}_{q} \boldsymbol{x}_{m}$$

query vector for the token at position m

$$R_{\Theta}^{m} = \begin{bmatrix} \cos(m\theta) & -\sin(m\theta) \\ \sin(m\theta) & \cos(m\theta) \end{bmatrix}$$

$$R_{\Theta}^{n} \boldsymbol{W}_{k} \boldsymbol{x}_{n}$$

key vector for the token at position n



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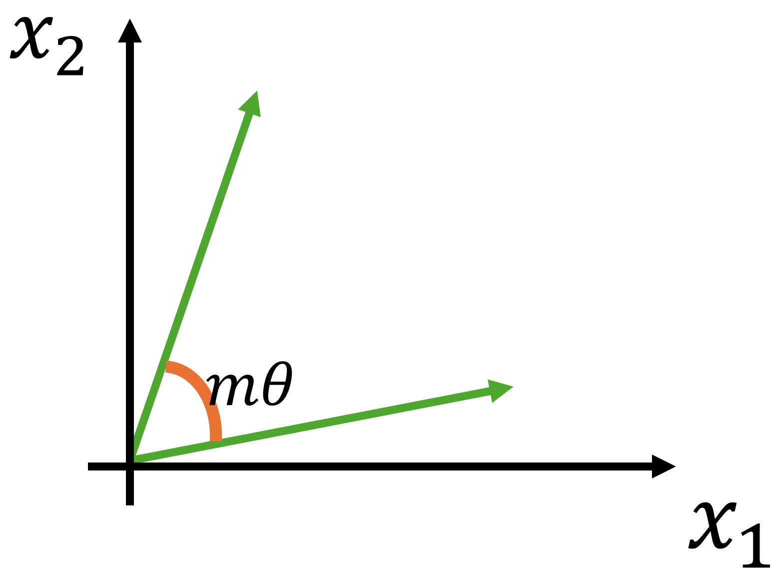

$$\alpha_{m,n} = \boldsymbol{x}_n^\top \boldsymbol{W}_k^\top (R_\Theta^n)^\top R_\Theta^m \boldsymbol{W}_q \boldsymbol{x}_m$$

Slide thanks to Jiabin Huang

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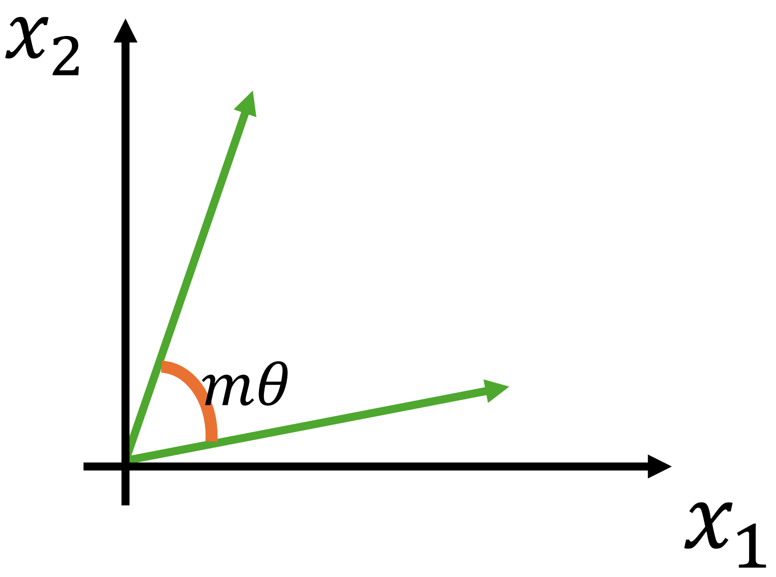

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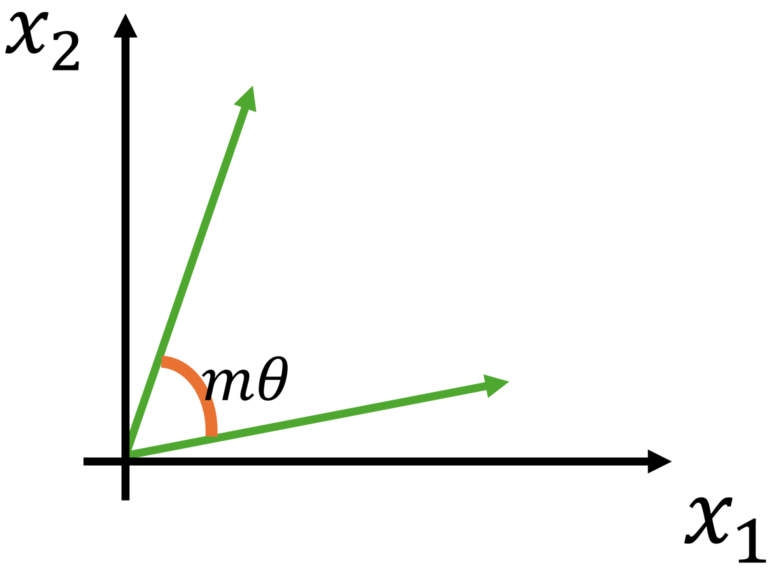

$$\alpha_{m,n} = \boldsymbol{x}_n^{\top}\,\boldsymbol{W}_k^{\top}(R_{\Theta}^{n})^{\top}R_{\Theta}^{m}\,\boldsymbol{W}_q\,\boldsymbol{x}_m$$

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$$R_{\Theta}^{m}\, \boldsymbol{W}_q \boldsymbol{x}_m$$

query vector for the token at position m

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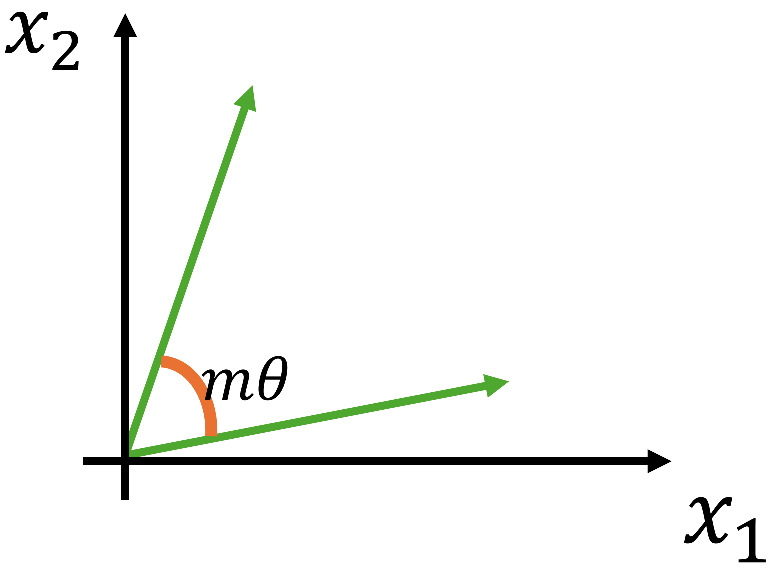
key vector for the token at position n



$$\alpha_{m,n} = \boldsymbol{x}_n^{\top} \boldsymbol{W}_k^{\top} R_{\Theta}^{-n} R_{\Theta}^{m} \boldsymbol{W}_q \boldsymbol{x}_m$$

Slide thanks to Jiabin Huang

$$R_{\Theta}^{m}\, \boldsymbol{W}_q\, \boldsymbol{x}_m$$

query vector for the token at position m

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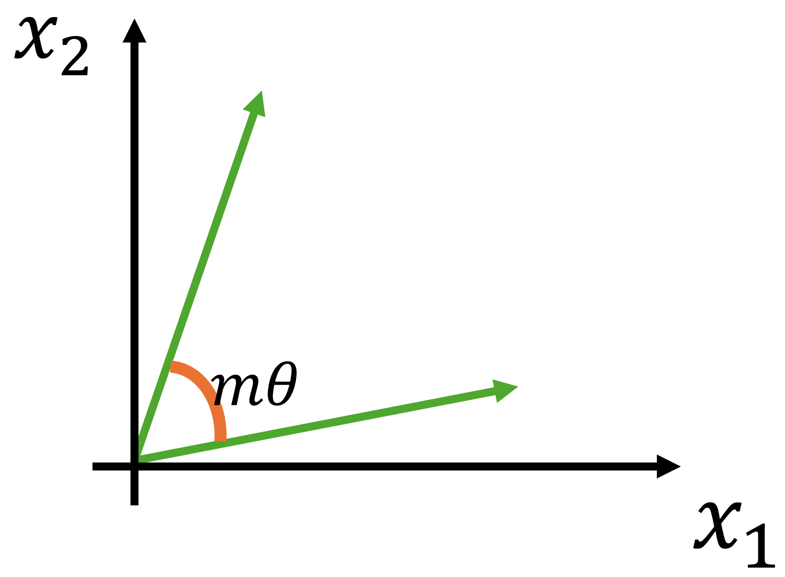
key vector for the token at position n



$$\alpha_{m,n} = \boldsymbol{x}_n^{\top}\, \boldsymbol{W}_k^{\top}\, R_{\Theta}^{m-n}\, \boldsymbol{W}_q\, \boldsymbol{x}_m$$

Slide thanks to Jiabin Huang

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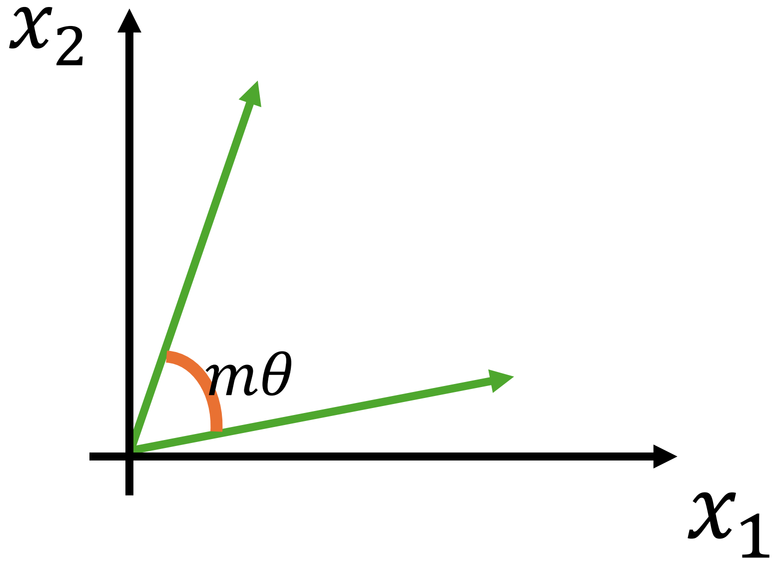

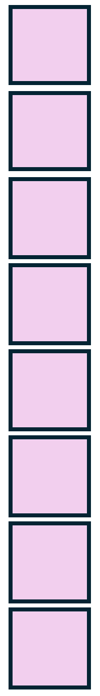

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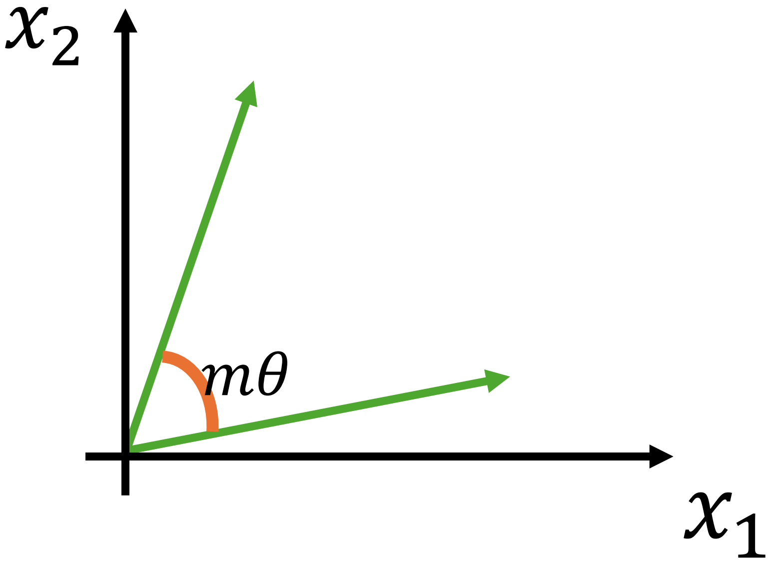

$$\begin{matrix}\square\\\square\end{matrix} = \begin{bmatrix} \cos(m\theta_1) & -\sin(m\theta_1) \\ \sin(m\theta_1) & \cos(m\theta_1) \end{bmatrix} \begin{matrix}\square\\\square\end{matrix}$$

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$\boldsymbol{W}_q \boldsymbol{x}_m$

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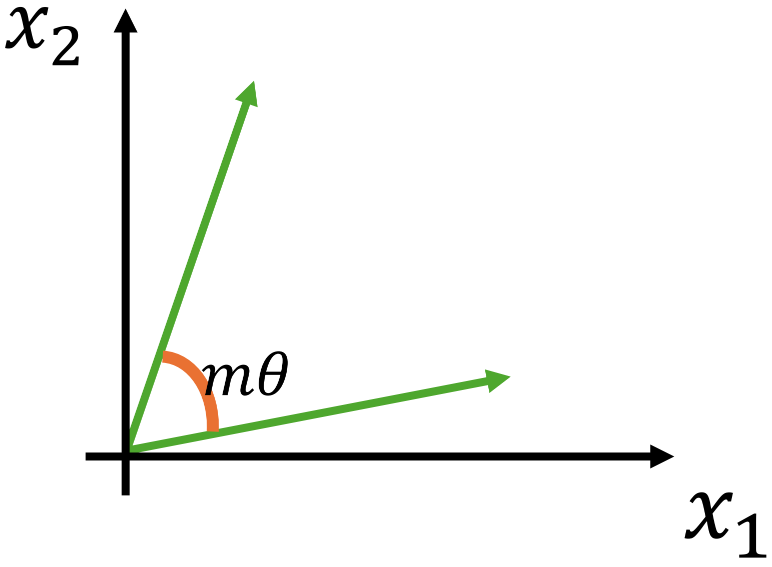

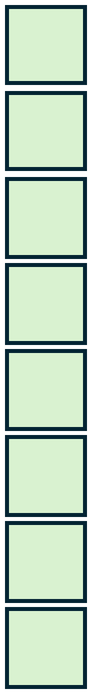

$$\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \cos(m\theta_1) & -\sin(m\theta_1) \\ \sin(m\theta_1) & \cos(m\theta_1) \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$$\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \cos(m\theta_2) & -\sin(m\theta_2) \\ \sin(m\theta_2) & \cos(m\theta_2) \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

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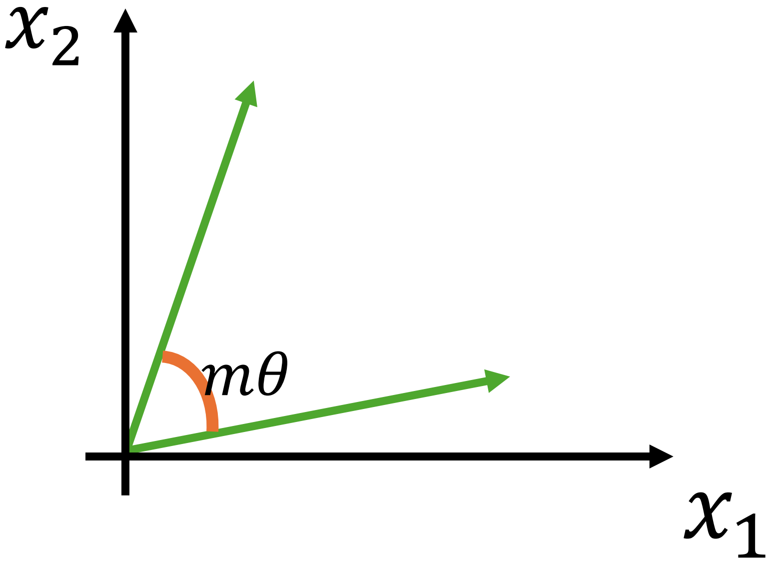
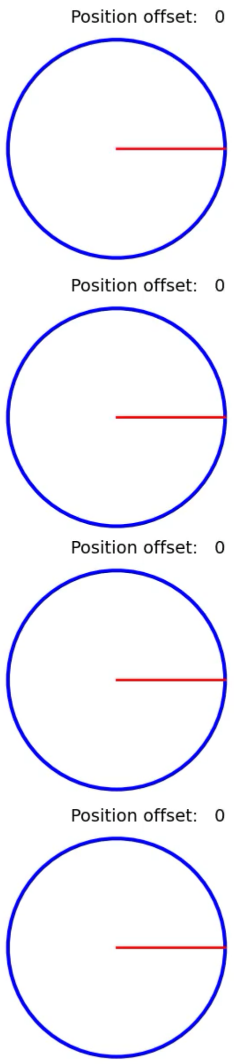
$f_q(\boldsymbol{x}_m, m)$ $\qquad$ $\boldsymbol{W}_q \boldsymbol{x}_m$

Slide thanks to Jiabin Huang

$$R_{\Theta}^{m} = \begin{bmatrix} \cos(m\theta) & -\sin(m\theta) \\ \sin(m\theta) & \cos(m\theta) \end{bmatrix}$$





$$= \begin{bmatrix} \cos(m\theta_1) & -\sin(m\theta_1) \\ \sin(m\theta_1) & \cos(m\theta_1) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(m\theta_2) & -\sin(m\theta_2) \\ \sin(m\theta_2) & \cos(m\theta_2) \end{bmatrix}$$

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$\boldsymbol{W}_q \boldsymbol{x}_m$

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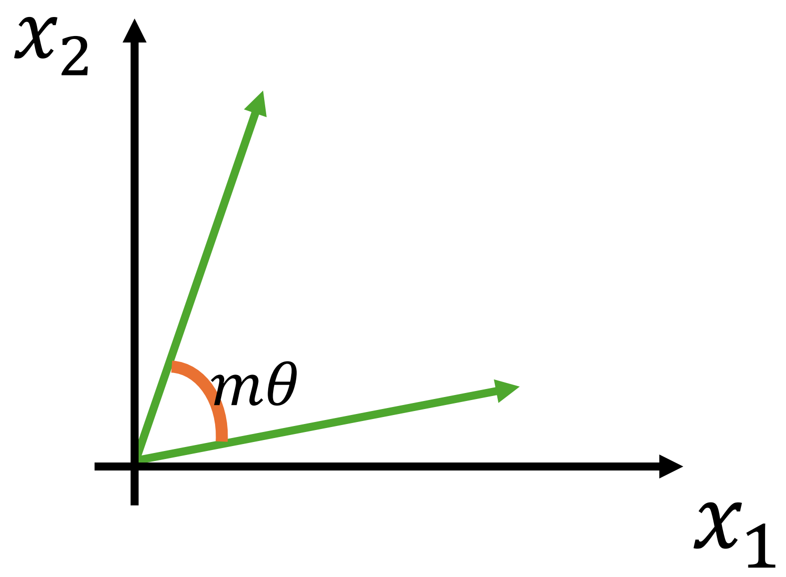
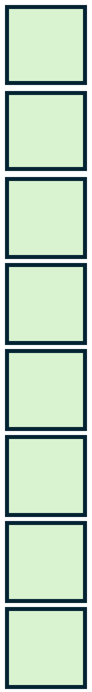
$$R_{\Theta}^{m} = \begin{bmatrix} \cos(m\theta) & -\sin(m\theta) \\ \sin(m\theta) & \cos(m\theta) \end{bmatrix}$$





$$\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \cos(m\theta_1) & -\sin(m\theta_1) \\ \sin(m\theta_1) & \cos(m\theta_1) \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$$\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \cos(m\theta_2) & -\sin(m\theta_2) \\ \sin(m\theta_2) & \cos(m\theta_2) \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

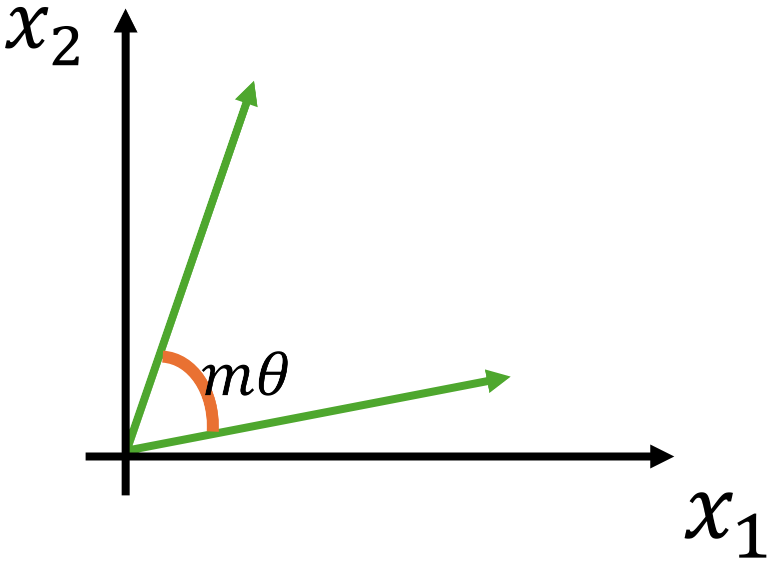
$$\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \cos(m\theta_3) & -\sin(m\theta_3) \\ \sin(m\theta_3) & \cos(m\theta_3) \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

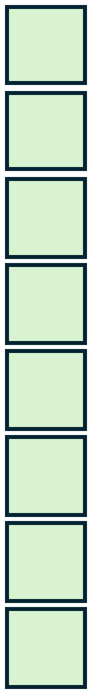
$$\begin{bmatrix} \square \\ \square \end{bmatrix} = \begin{bmatrix} \cos(m\theta_4) & -\sin(m\theta_4) \\ \sin(m\theta_4) & \cos(m\theta_4) \end{bmatrix} \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$f_q(\boldsymbol{x}_m, m)$

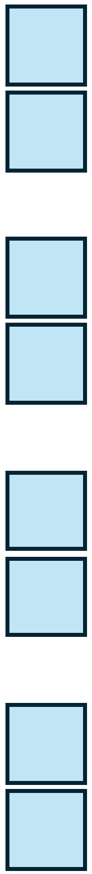
$\boldsymbol{W}_q \boldsymbol{x}_m$

Slide thanks to Jiabin Huang





$f_q(\boldsymbol{x}_m, m)$



$\boldsymbol{W}_q \boldsymbol{x}_m$

$$
\underbrace{}_{f_q(\boldsymbol{x}_m, m)} = \underbrace{\begin{pmatrix}
\cos m\theta_1 & -\sin m\theta_1 & 0 & 0 & \cdots & 0 & 0 \\
\sin m\theta_1 & \cos m\theta_1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cos m\theta_2 & -\sin m\theta_2 & \cdots & 0 & 0 \\
0 & 0 & \sin m\theta_2 & \cos m\theta_2 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \cos m\theta_l & -\sin m\theta_l \\
0 & 0 & 0 & 0 & \cdots & \sin m\theta_l & \cos m\theta_l
\end{pmatrix}}_{R^m_\Theta}
\underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ \vdots \\ q_8 \end{bmatrix}}_{W_q \boldsymbol{x}_m}
$$

Fast changing

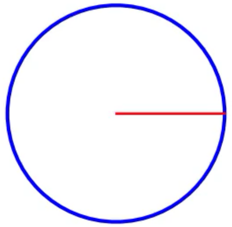
$$
f_q(\boldsymbol{x}_m, m) = \begin{bmatrix} \cos(m\theta_1) \\ \cos(m\theta_1) \\ \cos(m\theta_2) \\ \cos(m\theta_2) \\ \cos(m\theta_3) \\ \cos(m\theta_3) \\ \cos(m\theta_4) \\ \cos(m\theta_4) \end{bmatrix} \otimes \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} + \begin{bmatrix} \sin(m\theta_1) \\ \sin(m\theta_1) \\ \sin(m\theta_2) \\ \sin(m\theta_2) \\ \sin(m\theta_3) \\ \sin(m\theta_3) \\ \sin(m\theta_4) \\ \sin(m\theta_4) \end{bmatrix} \otimes \begin{bmatrix} -q_2 \\ q_1 \\ -q_4 \\ q_3 \\ -q_6 \\ q_5 \\ -q_8 \\ q_7 \end{bmatrix}
$$

Slowly changing

$$\theta_i = N^{-2i/d} \qquad N = 10{,}000$$

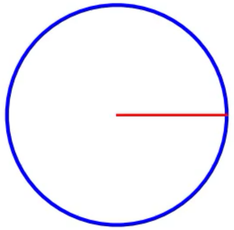
Slide thanks to Jiabin Huang

Position offset: 0



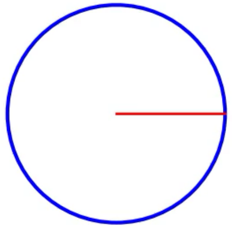
$$\begin{bmatrix}\square\\ \square\end{bmatrix} = \begin{bmatrix}\cos(m\theta_1) & -\sin(m\theta_1)\\ \sin(m\theta_1) & \cos(m\theta_1)\end{bmatrix}\begin{bmatrix}\square\\ \square\end{bmatrix}$$

Position offset: 0



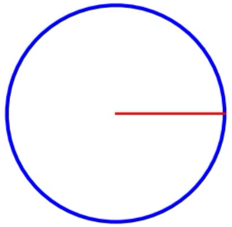
$$\begin{bmatrix}\square\\ \square\end{bmatrix} = \begin{bmatrix}\cos(m\theta_2) & -\sin(m\theta_2)\\ \sin(m\theta_2) & \cos(m\theta_2)\end{bmatrix}\begin{bmatrix}\square\\ \square\end{bmatrix}$$

Position offset: 0



$$\begin{bmatrix}\square\\ \square\end{bmatrix} = \begin{bmatrix}\cos(m\theta_3) & -\sin(m\theta_3)\\ \sin(m\theta_3) & \cos(m\theta_3)\end{bmatrix}\begin{bmatrix}\square\\ \square\end{bmatrix}$$

Position offset: 0



$$\begin{bmatrix}\square\\ \square\end{bmatrix} = \begin{bmatrix}\cos(m\theta_4) & -\sin(m\theta_4)\\ \sin(m\theta_4) & \cos(m\theta_4)\end{bmatrix}\begin{bmatrix}\square\\ \square\end{bmatrix}$$

$f_q(\boldsymbol{x}_m, m)$ $\qquad\qquad$ $\boldsymbol{W}_q\boldsymbol{x}_m$

Slide thanks to Jiabin Huang

2D ROPE

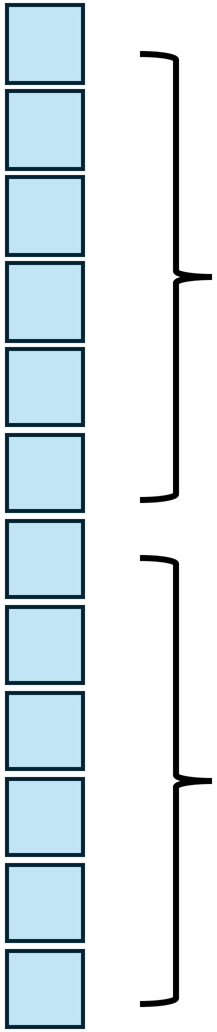
Patch (m, n)

$$
\begin{bmatrix}
\cos(\theta_1 x) & -\sin(\theta_1 x) & 0 & 0 & \dots & 0 & 0 \\
\sin(\theta_1 x) & \cos(\theta_1 x) & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & \cos(\theta_2 x) & -\sin(\theta_2 x) & \dots & 0 & 0 \\
0 & 0 & \sin(\theta_2 x) & \cos(\theta_2 x) & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & \cos(\theta_l x) & -\sin(\theta_l x) \\
0 & 0 & 0 & & \dots & \sin(\theta_l x) & \cos(\theta_l x)
\end{bmatrix}
$$

$$R_x$$

$$
\begin{bmatrix}
\cos(\theta_1 y) & -\sin(\theta_1 y) & 0 & 0 & \dots & 0 & 0 \\
\sin(\theta_1 y) & \cos(\theta_1 y) & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & \cos(\theta_2 y) & -\sin(\theta_2 y) & \dots & 0 & 0 \\
0 & 0 & \sin(\theta_2 y) & \cos(\theta_2 y) & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & \cos(\theta_l y) & -\sin(\theta_l y) \\
0 & 0 & 0 & 0 & \dots & \sin(\theta_l y) & \cos(\theta_l y)
\end{bmatrix}
$$

$$R_y$$



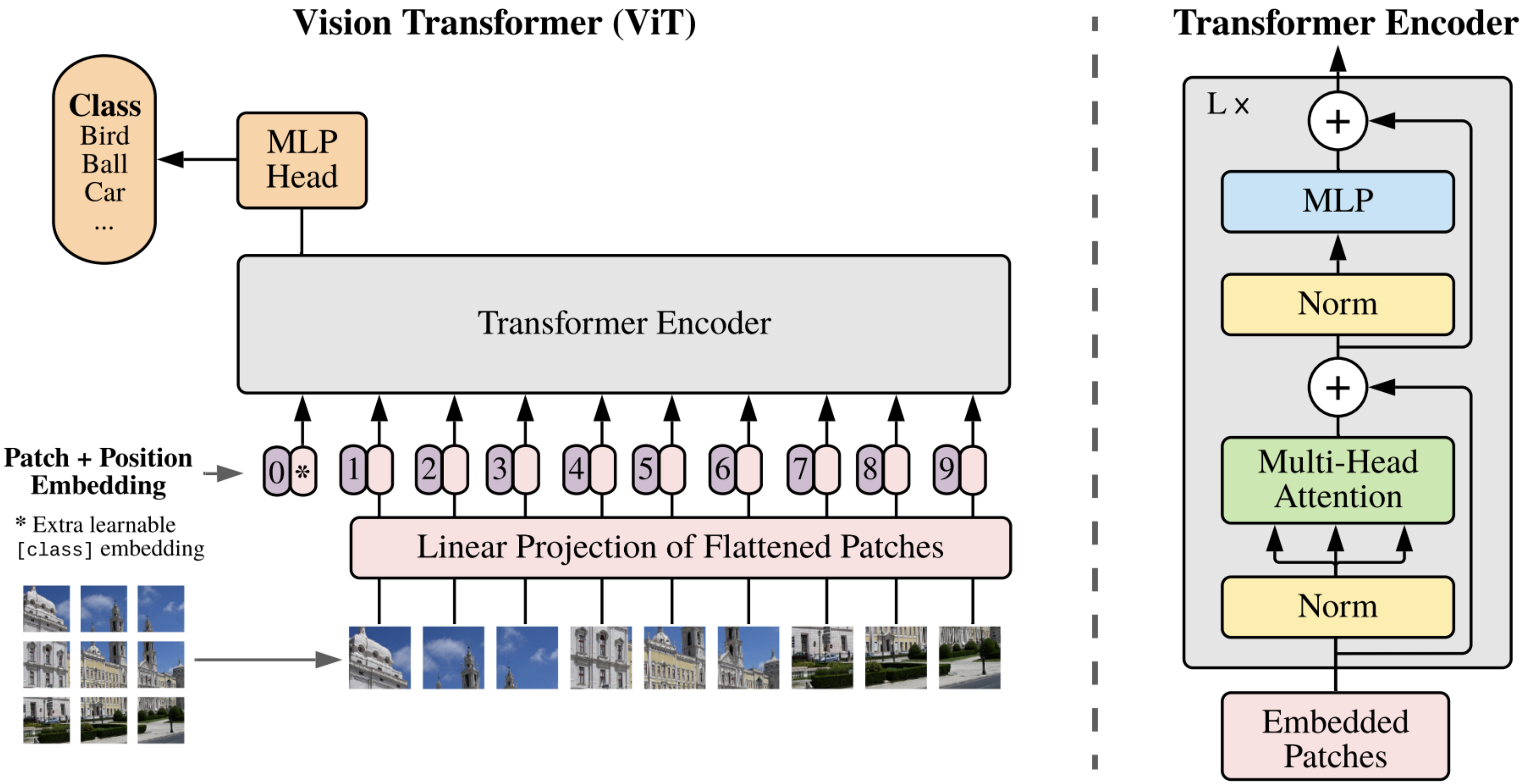
Rope on X values

Rope on Y values

Switching Gears

Using ViT!

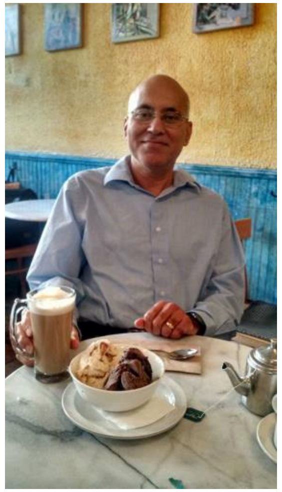
Vision Transformer



An image is worth 16x16 words, Dosovitskiy e tal. ICLR 2021

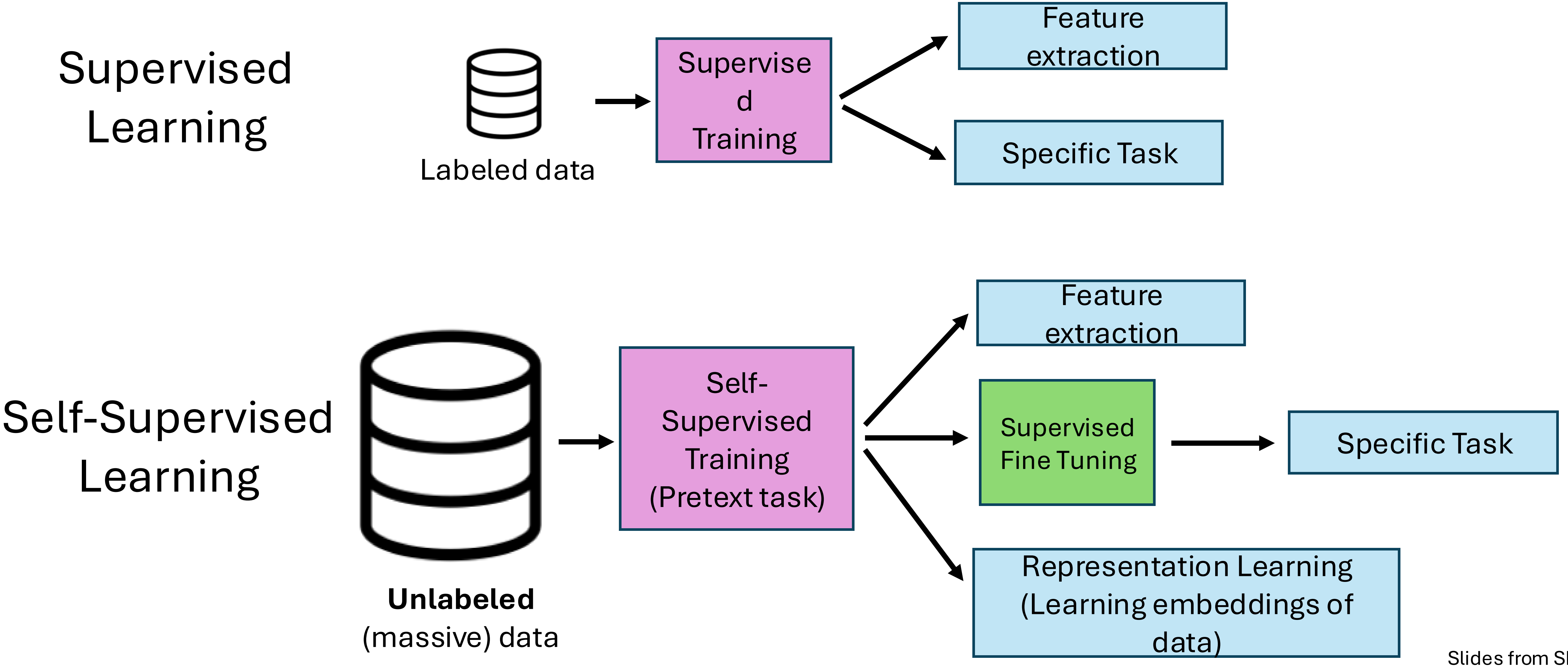
Is semantic supervision necessary to learn good representations?

- Manual labeling doesn't scale, suffers from biases
- Plenty of unlabeled visual data already, and growing really fast
- *And subject of the Gelato Bet:*
- *Made on Sept 23 2014*
- *If, by the first day of autumn (Sept 23) of 2015, a method will exist that can match or beat the performance of R-CNN on Pascal VOC detection, without the use of any extra, human annotations (e.g. ImageNet) as pre-training, Mr. Malik promises to buy Mr. Efros one (1) gelato (2 scoops: one chocolate, one vanilla).*



The Gelato Bet

Self supervised learning with ViTs



Slides from Shai Bagon

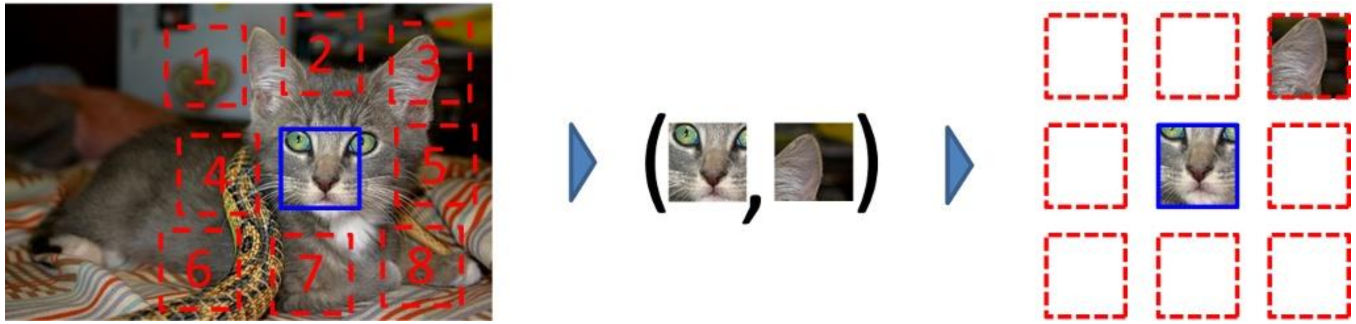
Pre-train representations on a pre-text task

E.g. Colorization

After pre-training, use representation for down-stream tasks.

Many other possibilities,

- Spatial relationship between pair of patches



- Predict sound / frame ordering in a video
- Encourage two augmentations of same image to be closer to each other than to another image
- Predict hidden image patches from context

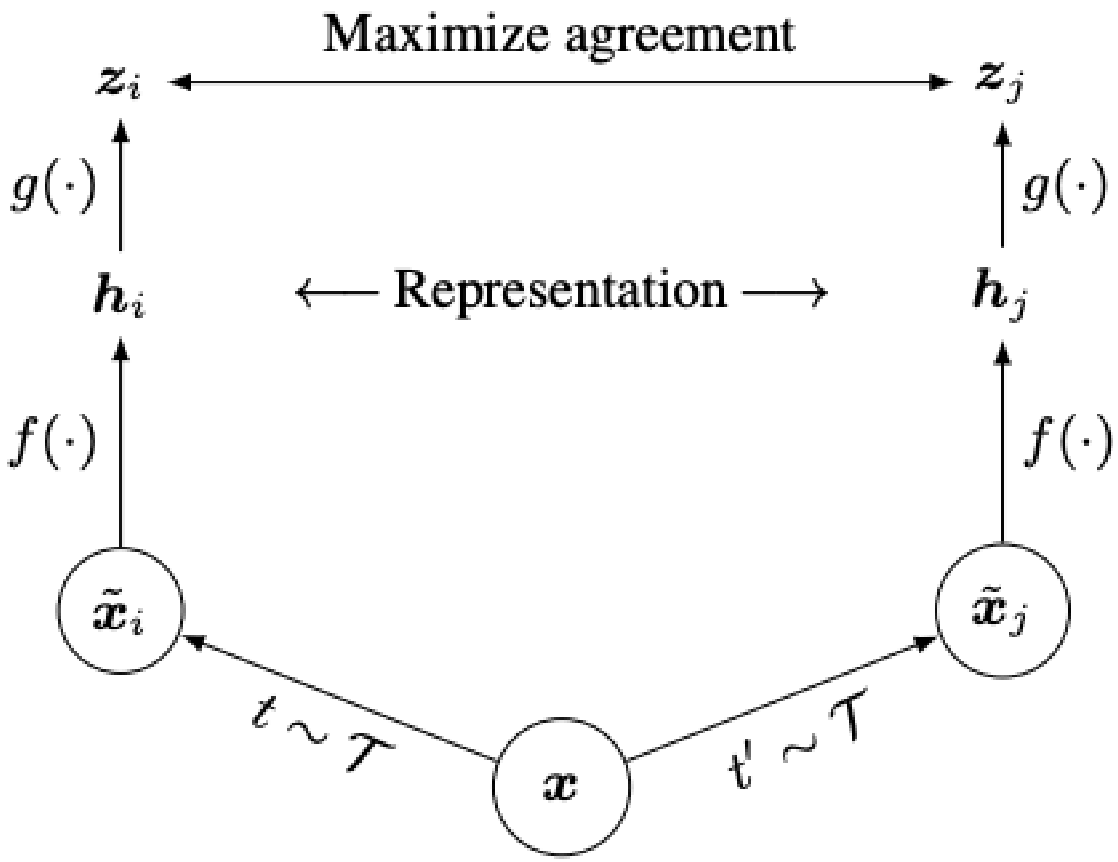
[Split-Brain Autoencoders: Unsupervised Learning by Cross-Channel Prediction], Zhang et al. CVPR 2017
[Context as Supervisory Signal: Discovering Objects with Predictable Context], Doersch et al. ICCV 2015

Slide credit Saurabh Gupta

Contrastive Learning

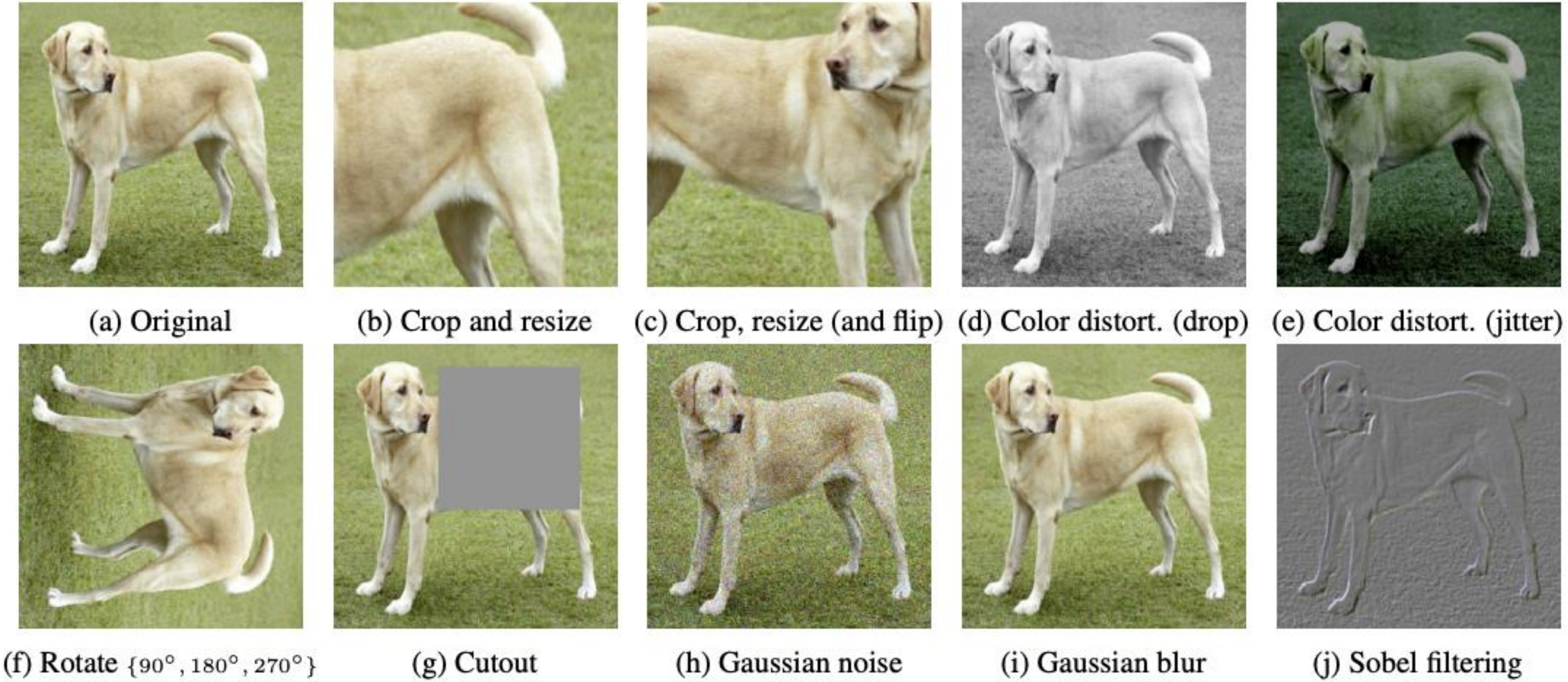
- Encourage two augmentations of an image to be close.
- Using a contrastive loss:

$$\ell_{i,j} = -\log \frac{\exp(\mathrm{sim}(\boldsymbol{z}_i, \boldsymbol{z}_j)/\tau)}{\sum_{k=1}^{2N} \mathbb{1}_{[k \neq i]} \exp(\mathrm{sim}(\boldsymbol{z}_i, \boldsymbol{z}_k)/\tau)}$$



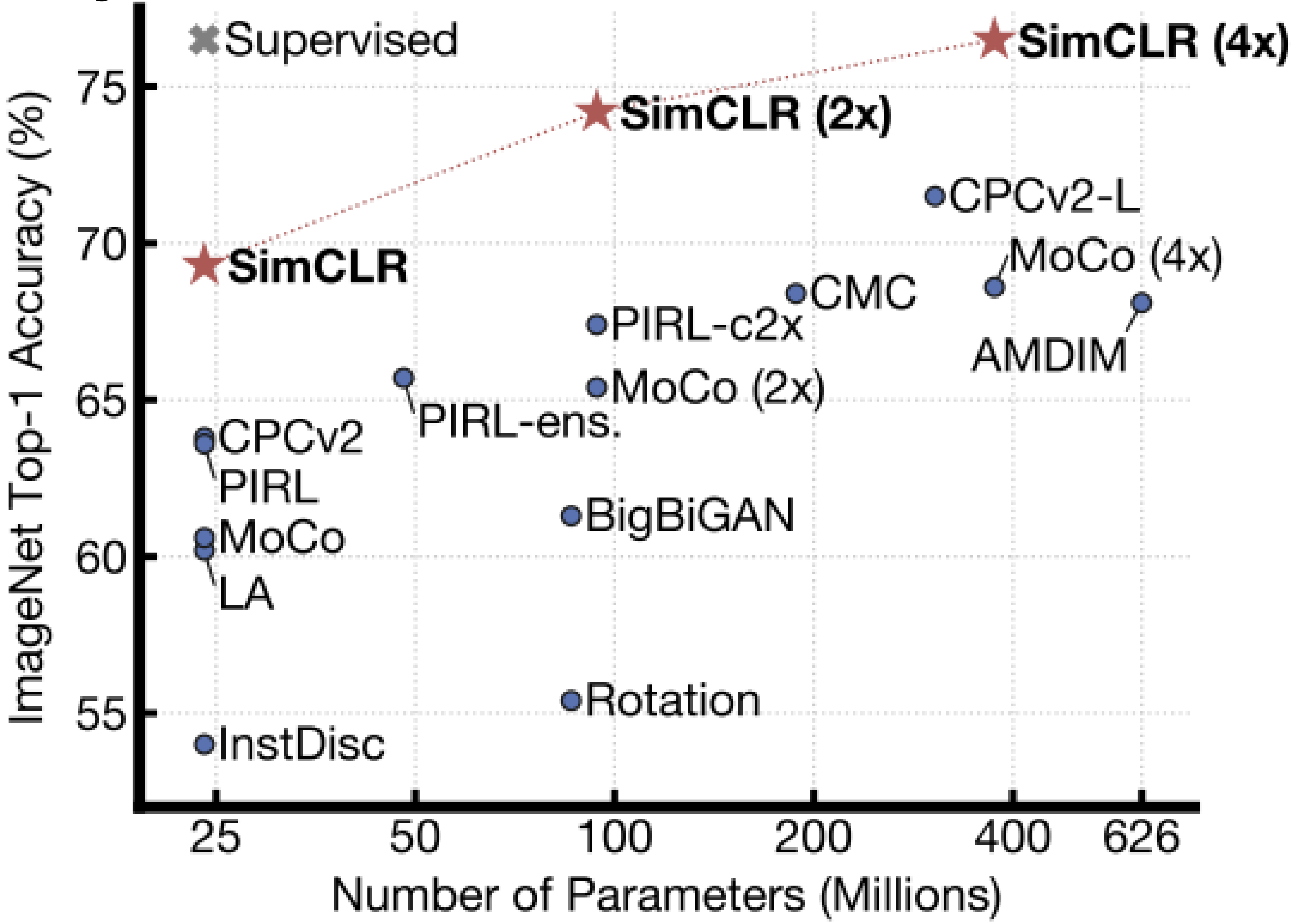
A Simple Framework for Contrastive Learning of Visual Representations, Chen et al. ICML 2020
See also: Momentum Contrast for Unsupervised Visual Representation Learning, He et al. CVPR 2020

Augmentations



[A Simple Framework for Contrastive Learning of Visual Representations](), Chen et al. ICML 2020

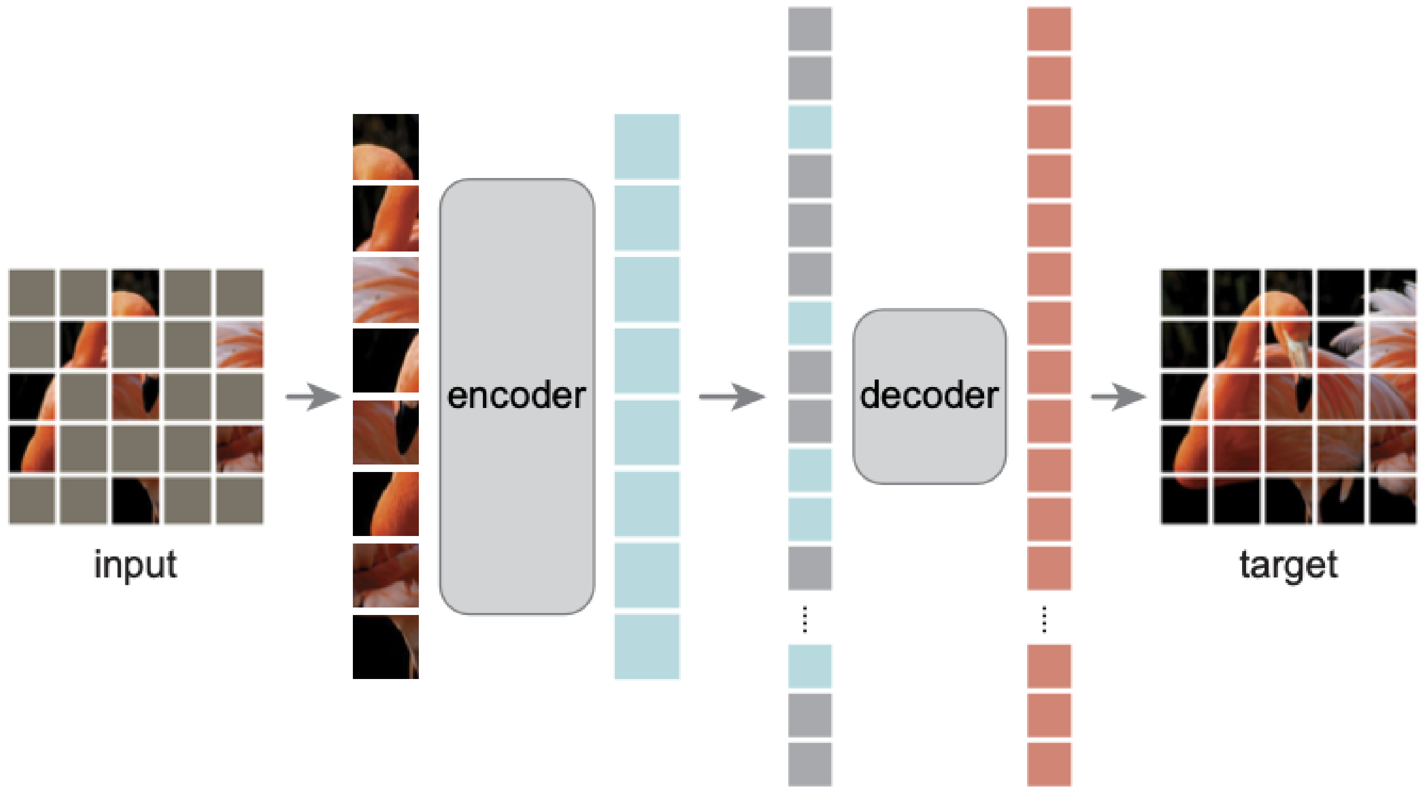
Was a very active area



[A Simple Framework for Contrastive Learning of Visual Representations](), Chen et al. ICML 2020

Masked Auto-Encoders

- Mask out image patches, predict masked patches from visible patches.
- Pre-train encoder & decoder.
- Use encoder as an image representation.
- Used as frozen backbone for various vision tasks like detection/segmentation (later in class)



Masked Autoencoders Are Scalable Vision Learners, He et al. CVPR 2022

Slide credit Saurabh Gupta

Better than semantic supervision on ImageNet 1K!

method	pre-train data	AP^{box} ViT-B	AP^{box} ViT-L	AP^{mask} ViT-B	AP^{mask} ViT-L
supervised	IN1K w/ labels	47.9	49.3	42.9	43.9
MoCo v3	IN1K	47.9	49.3	42.7	44.0
BEiT	IN1K+DALLE	49.8	**53.3**	44.4	47.1
MAE	IN1K	**50.3**	**53.3**	**44.9**	**47.2**

Table 4. **COCO object detection and segmentation** using a ViT Mask R-CNN baseline. All entries are based on our implementation. Self-supervised entries use IN1K data *without* labels. Mask AP follows a similar trend as box AP.

method	pre-train data	ViT-B	ViT-L
supervised	IN1K w/ labels	47.4	49.9
MoCo v3	IN1K	47.3	49.1
BEiT	IN1K+DALLE	47.1	53.3
MAE	IN1K	**48.1**	**53.6**

Table 5. **ADE20K semantic segmentation** (mIoU) using UperNet. BEiT results are reproduced using the official code. Other entries are based on our implementation. Self-supervised entries use IN1K data *without* labels.

Scaling behavior →

dataset	ViT-B	ViT-L	ViT-H	ViT-H_{448}	prev best
iNat 2017	70.5	75.7	79.3	**83.4**	75.4 [55]
iNat 2018	75.4	80.1	83.0	**86.8**	81.2 [54]
iNat 2019	80.5	83.4	85.7	**88.3**	84.1 [54]
Places205	63.9	65.8	65.9	**66.8**	66.0 [19]†
Places365	57.9	59.4	59.8	**60.3**	58.0 [40]‡

Table 6. **Transfer learning accuracy on classification datasets**, using MAE pre-trained on IN1K and then fine-tuned. We provide system-level comparisons with the previous best results.

†: pre-trained on 1 billion images. ‡: pre-trained on 3.5 billion images.

Masked Autoencoders Are Scalable Vision Learners, He et al. CVPR 2022

Slides credit Saurabh Gupta

Improves performance on ImageNet itself

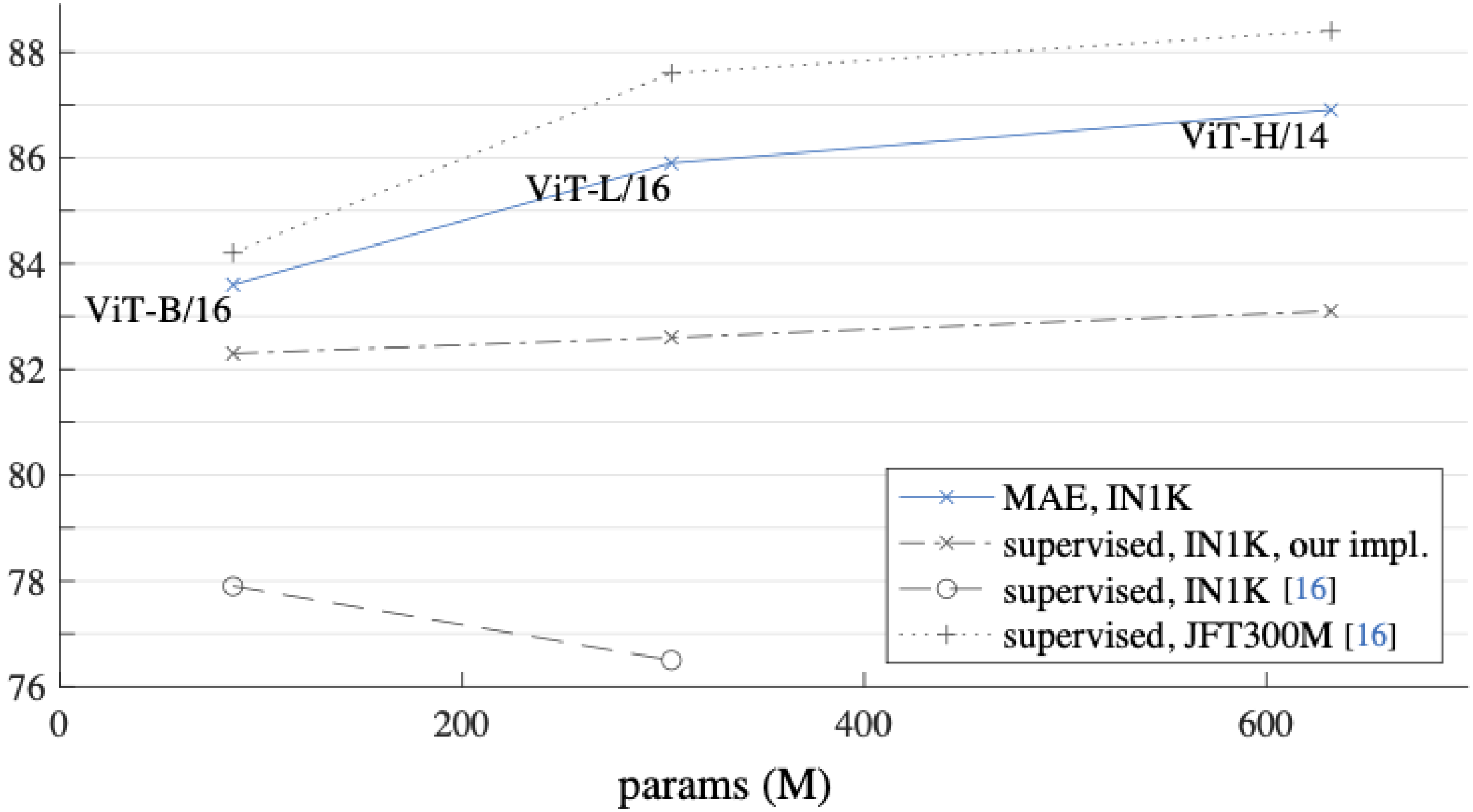


Figure 8. **MAE pre-training *vs*. supervised pre-training**, evaluated by fine-tuning in ImageNet-1K (224 size). We compare with the original ViT results [16] trained in IN1K or JFT300M.

Masked Autoencoders Are Scalable Vision Learners, He et al. CVPR 2022

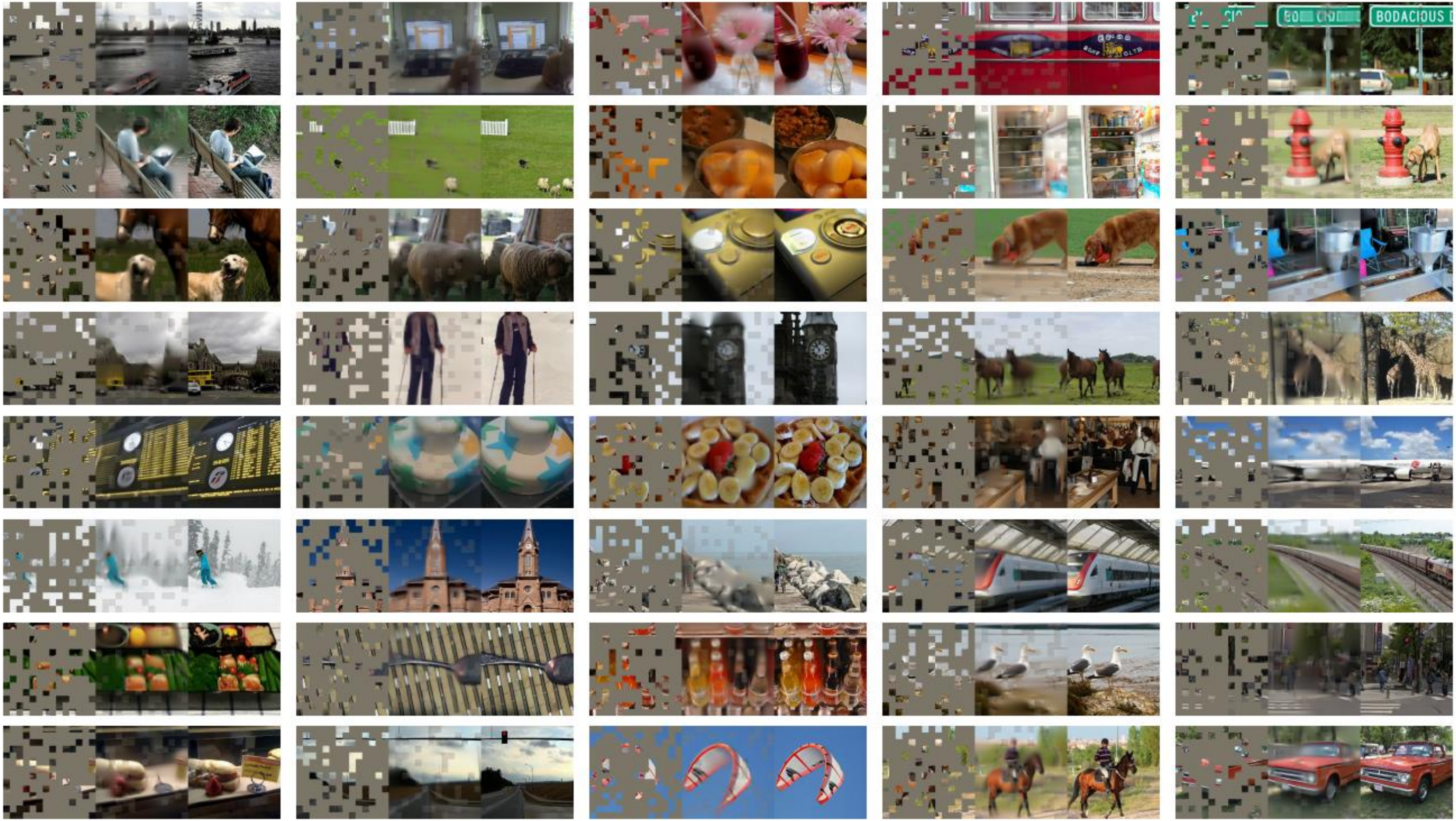
Slides from Saurabh Gupta

Better than past self-supervision approaches

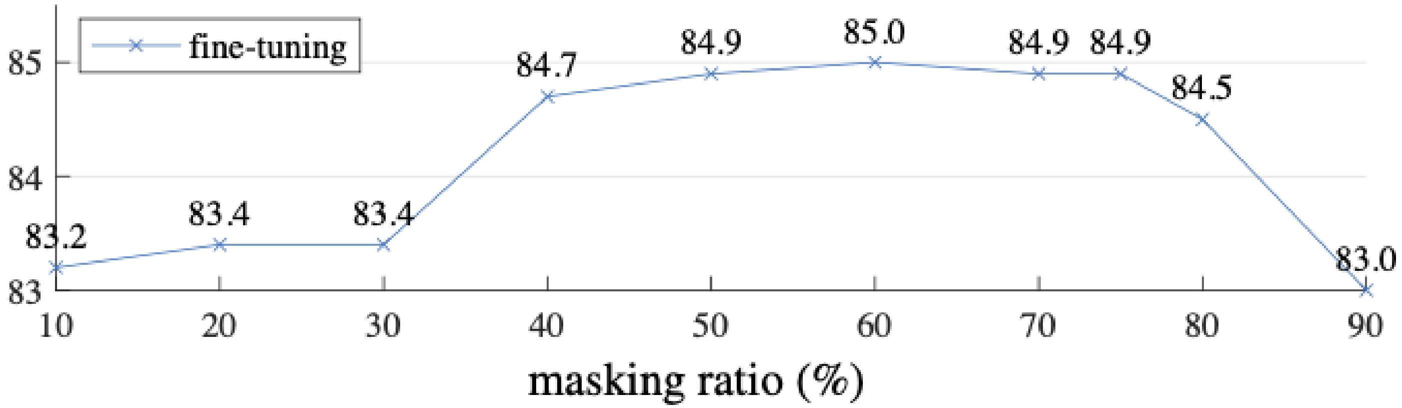
method	pre-train data	ViT-B	ViT-L	ViT-H	ViT-H_{448}
scratch, our impl.	-	82.3	82.6	83.1	-
DINO [5]	IN1K	82.8	-	-	-
MoCo v3 [9]	IN1K	83.2	84.1	-	-
BEiT [2]	IN1K+DALLE	83.2	85.2	-	-
MAE	IN1K	<u>83.6</u>	<u>85.9</u>	<u>86.9</u>	**87.8**

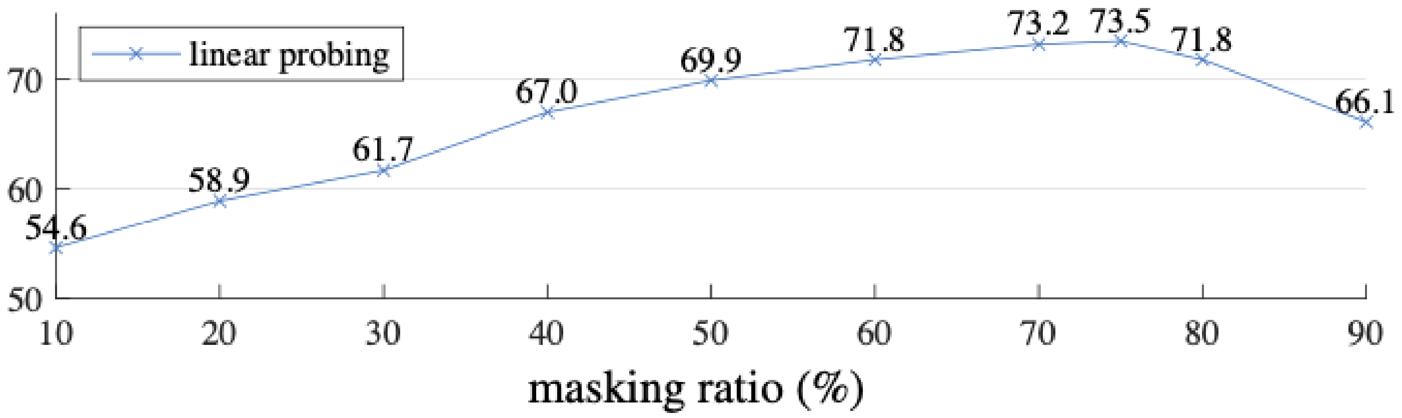
Masked Autoencoders Are Scalable Vision Learners, He et al. CVPR 2022

Slides from Saurabh Gupta


BODACIOUS

Ablations





Need high masking ratio for good learning.
NLP models use 15-20% masking ratio.

case	ft	lin	FLOPs
encoder w/ [M]	84.2	59.6	3.3×
encoder w/o [M]	**84.9**	**73.5**	**1**×

Faster and better to not input masked out patches to encoder

case	ft	lin
pixel (w/o norm)	84.9	73.5
pixel (w/ norm)	**85.4**	**73.9**
PCA	84.6	72.3
dVAE token	85.3	71.6

Normalized pixels are a better target than discrete tokens / PCA coefficients

Masked Autoencoders Are Scalable Vision Learners, He et al. CVPR 2022

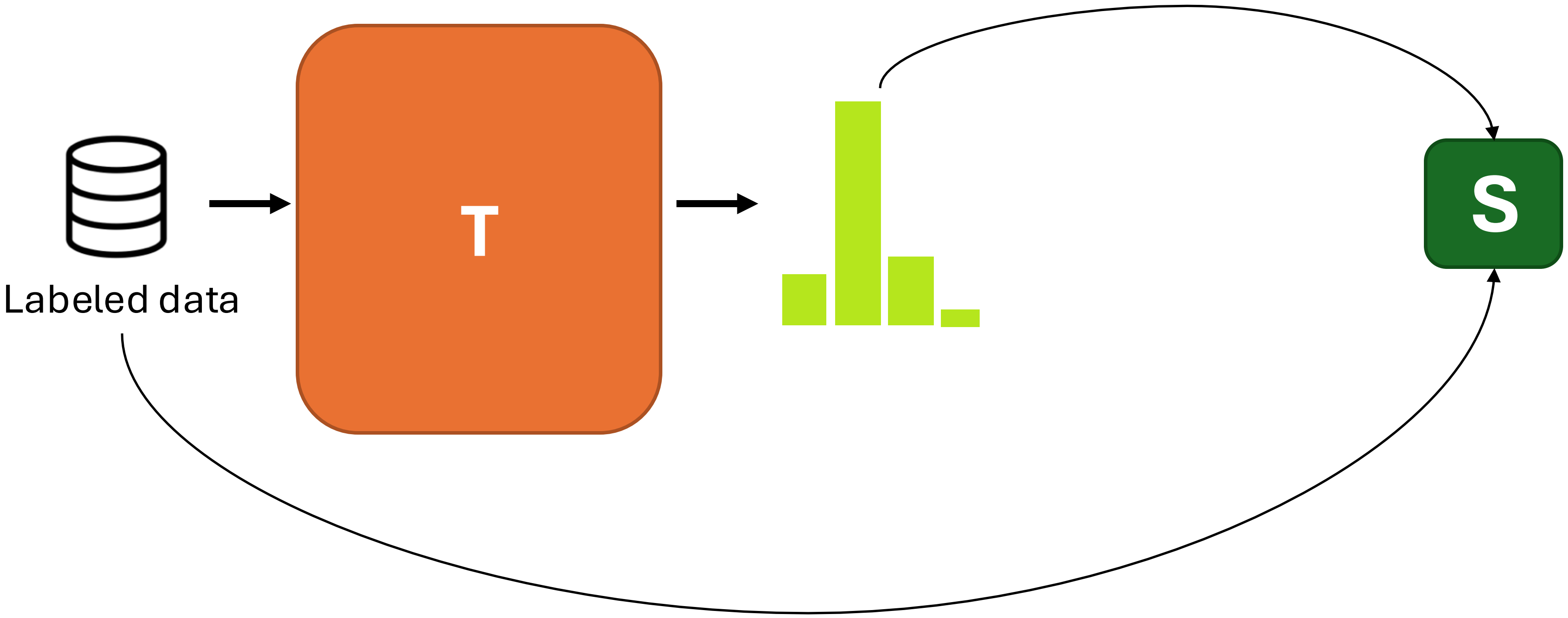
Slides from Saurabh Gupta

Take away

- Able to pre-train data-hungry ViT model
- Very well structured experiments! Read and learn from it.

DiNO - Approach

- Self supervised learning as a special case of **knowledge distillation**

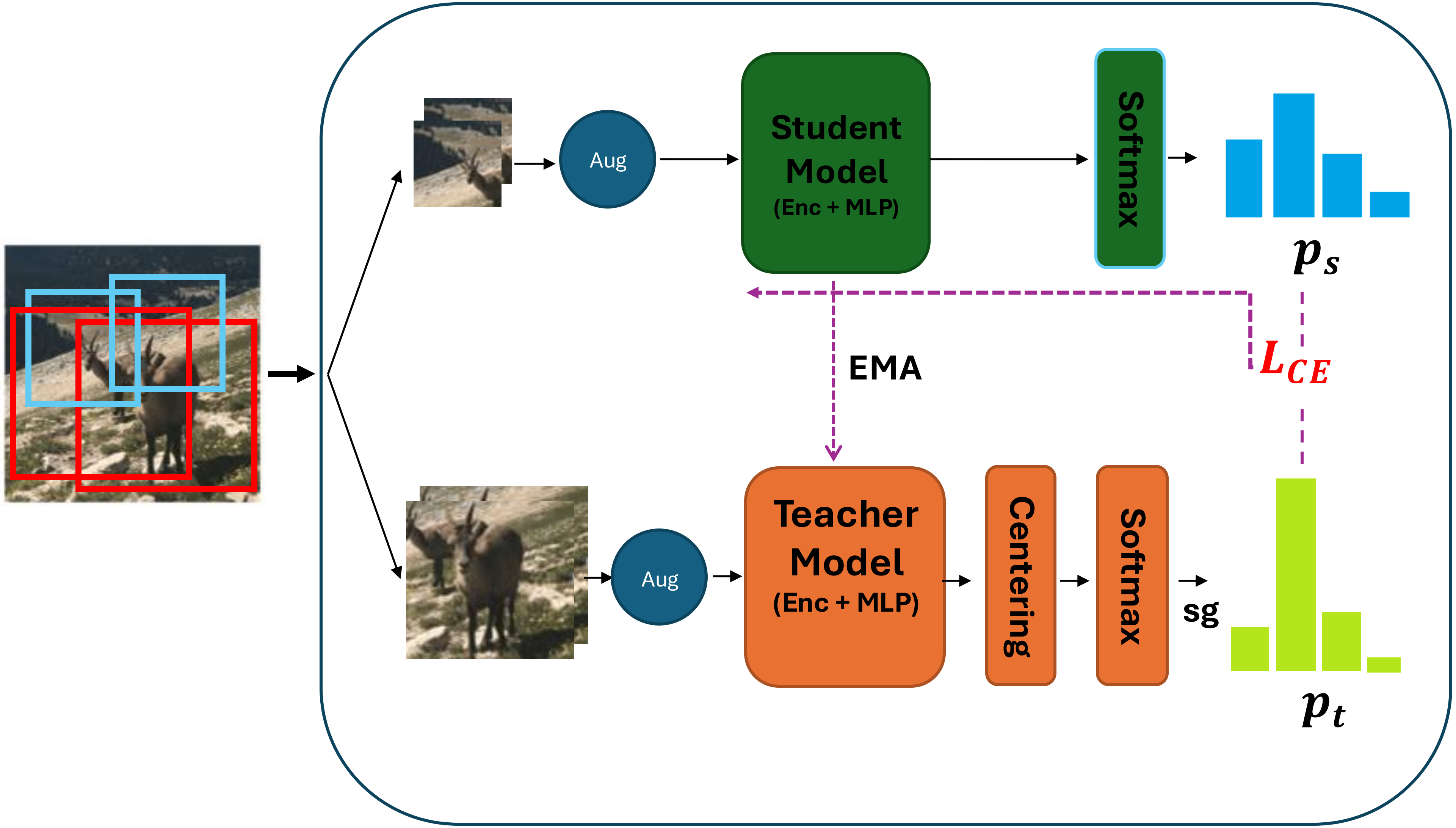


Slides from Shai Bagon

DiNO - Training

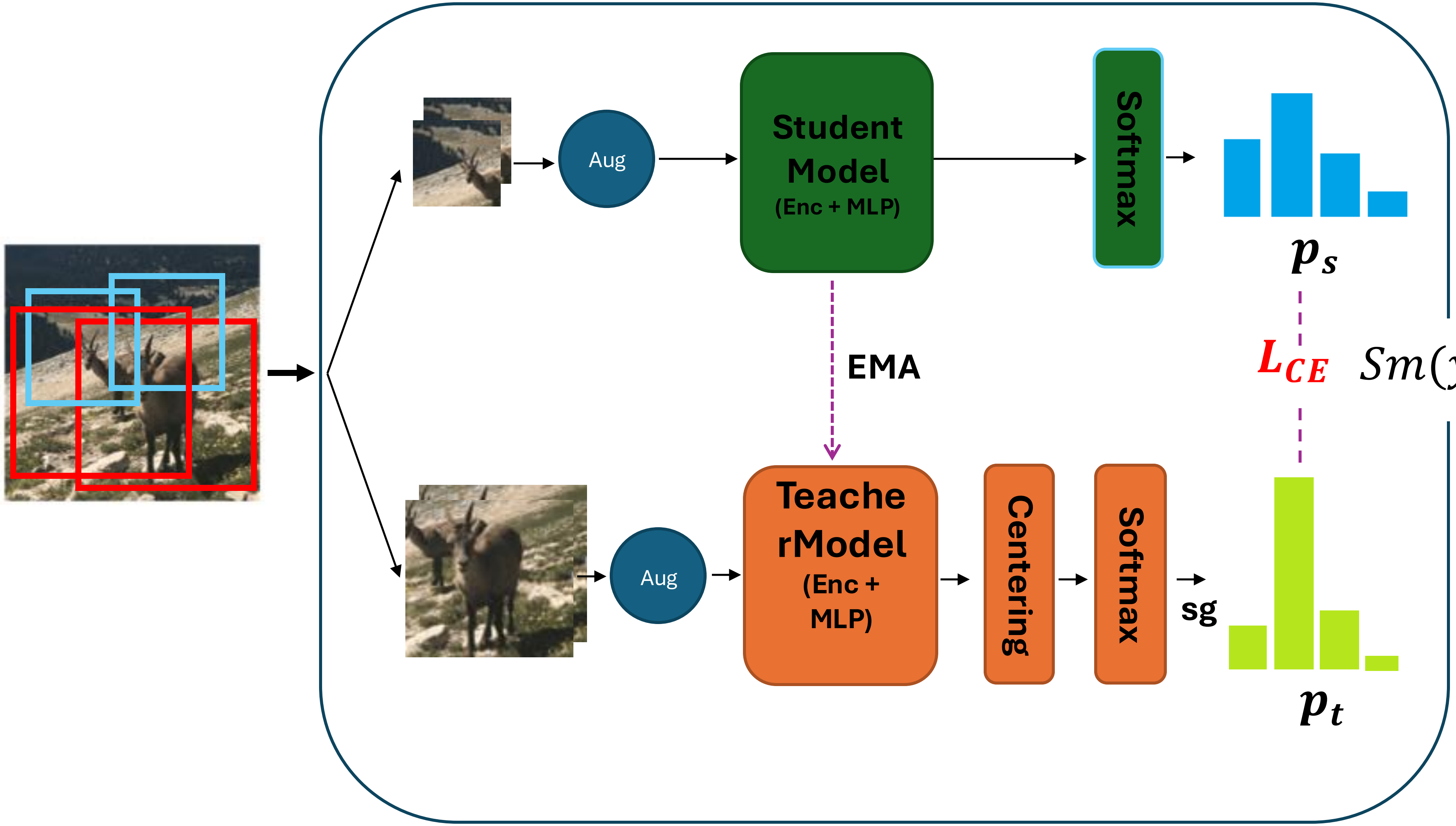
Slides from Shai Bagon

DiNO - Training



Slides from Shai Bagon

DiNO - Training



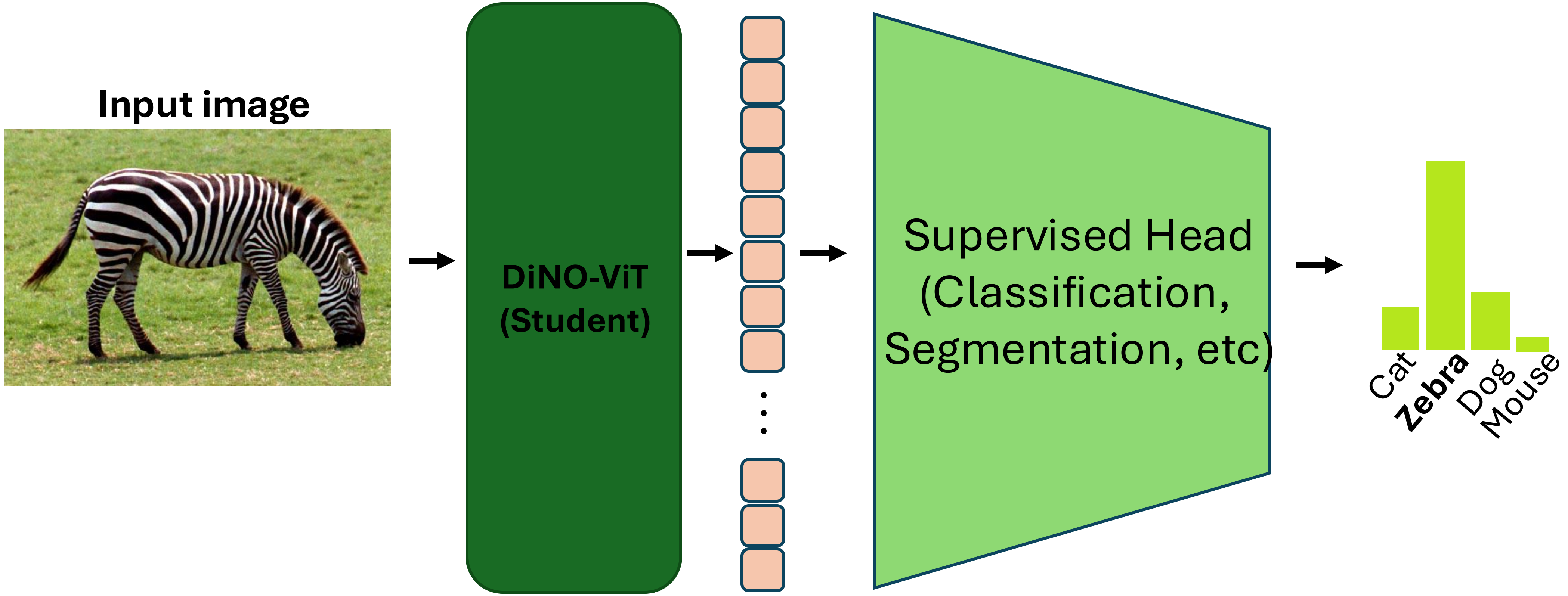
$$Sm(y) = \frac{exp(y_i/\tau)}{\sum_{j=1}^{K} exp(y_j/\tau)}$$

Slides from Shai Bagon

DiNO - Explanation

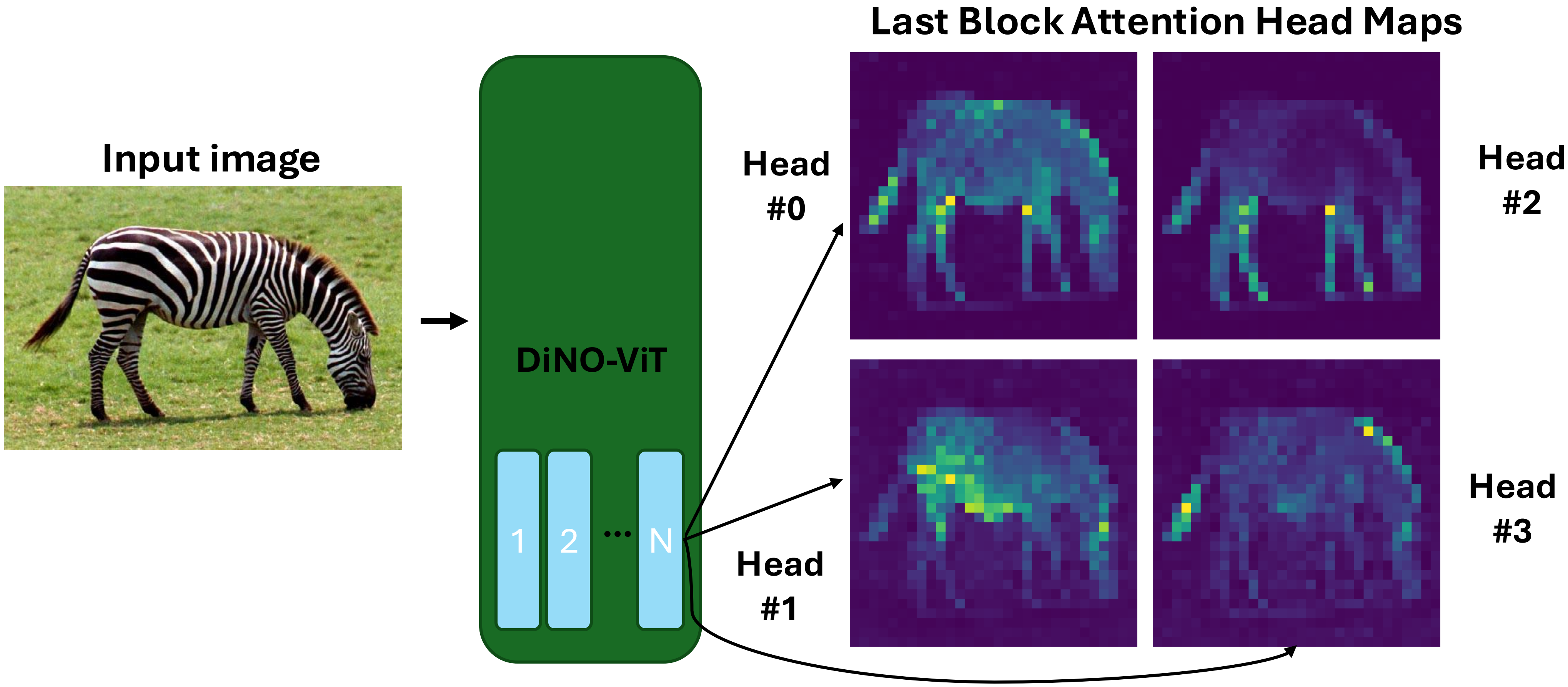
- Augmentations
  - Tells the model what to ignore
    - Collor jitter, Gaussian Blur, Solarize
    - Acts as a data prior
- “Global – local” cropping
- Teacher out-distribution sharpening via centering & Low-temperature in softmax
- The student encoder learns “abstract representations”
  - No awareness of “class labels” or meaning behind logits

DiNO - Inference #1



Slides from Shai Bagon

DiNO - Inference #2



Slides from Shai Bagon

DiNO - Inference #2

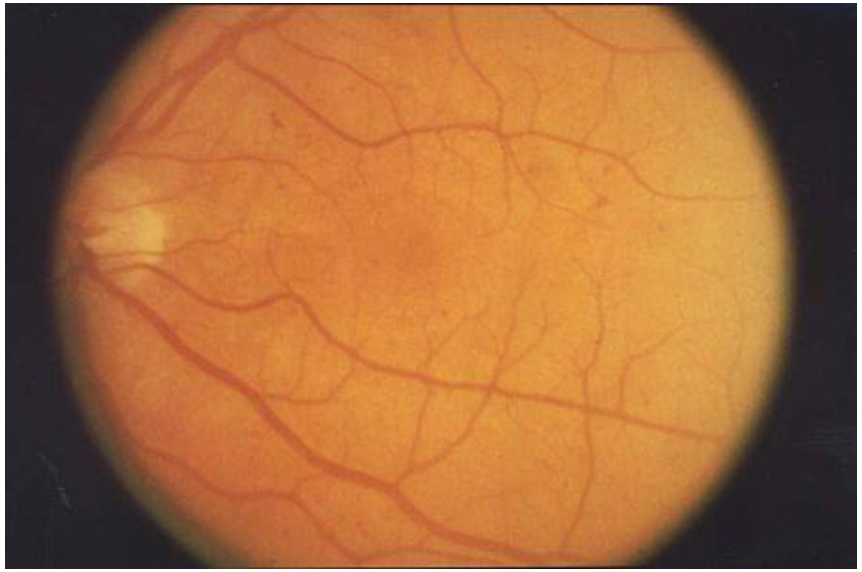
Input **Supervised Segmentation** **DiNO (SSL) Segmentation**

Slides from Shai Bagon

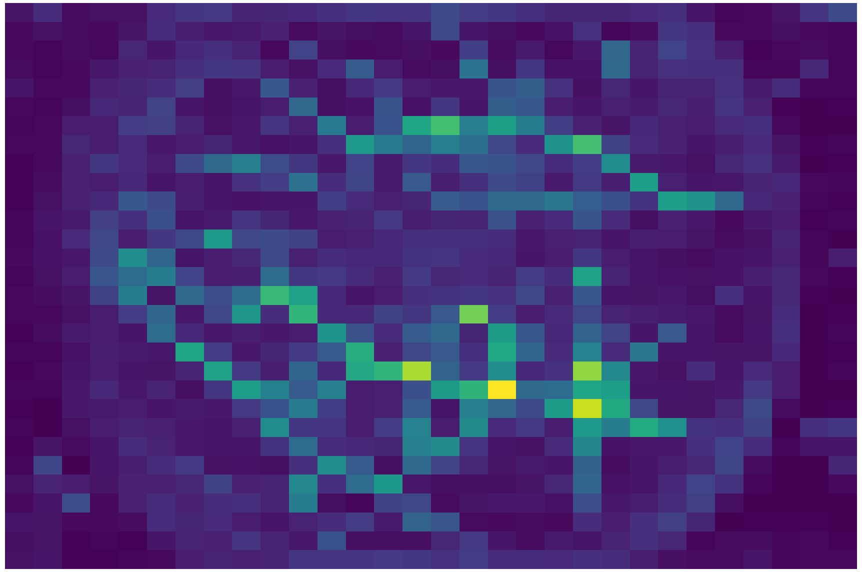
DiNO - Inference #2

- Self supervised learning also makes learned representations applicable to out-of-distribution data

Input image



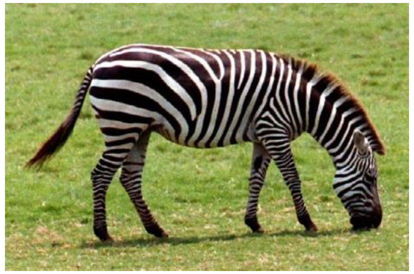
Last block attention map

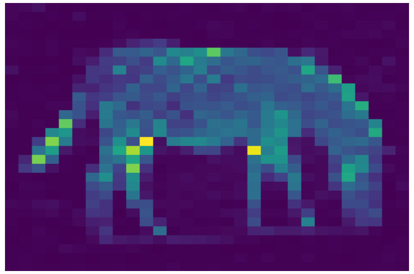


Slides from Shai Bagon

Usage

```
vit_model = torch.hub.load('facebookresearch/dino:main’,
                          f'dino_vits16', pretrained=True)
img = imread(‘zebra.png’)

x = vit_model.prepare_tokens(img)
for blk in vit_model.blocks[:-1]:
    x = blk(x)
attn_maps = vit_model.blocks[-1](x, return_attention=True)

# Choose head, Get attention map of class token
attn_map = attn_maps[0, HEAD, 0, 1:].reshape((1, 1, H_PATCHES, W_PATCHES))
attn_map = F.interpolate(attn_map, scale_factor=16, mode="nearest")
```

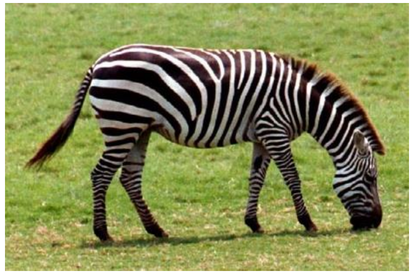


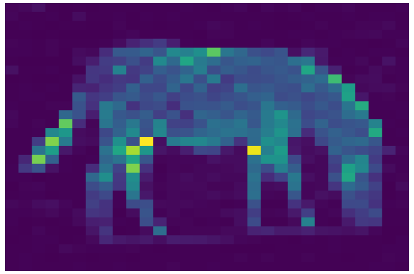


Slides from Shai Bagon

Usage

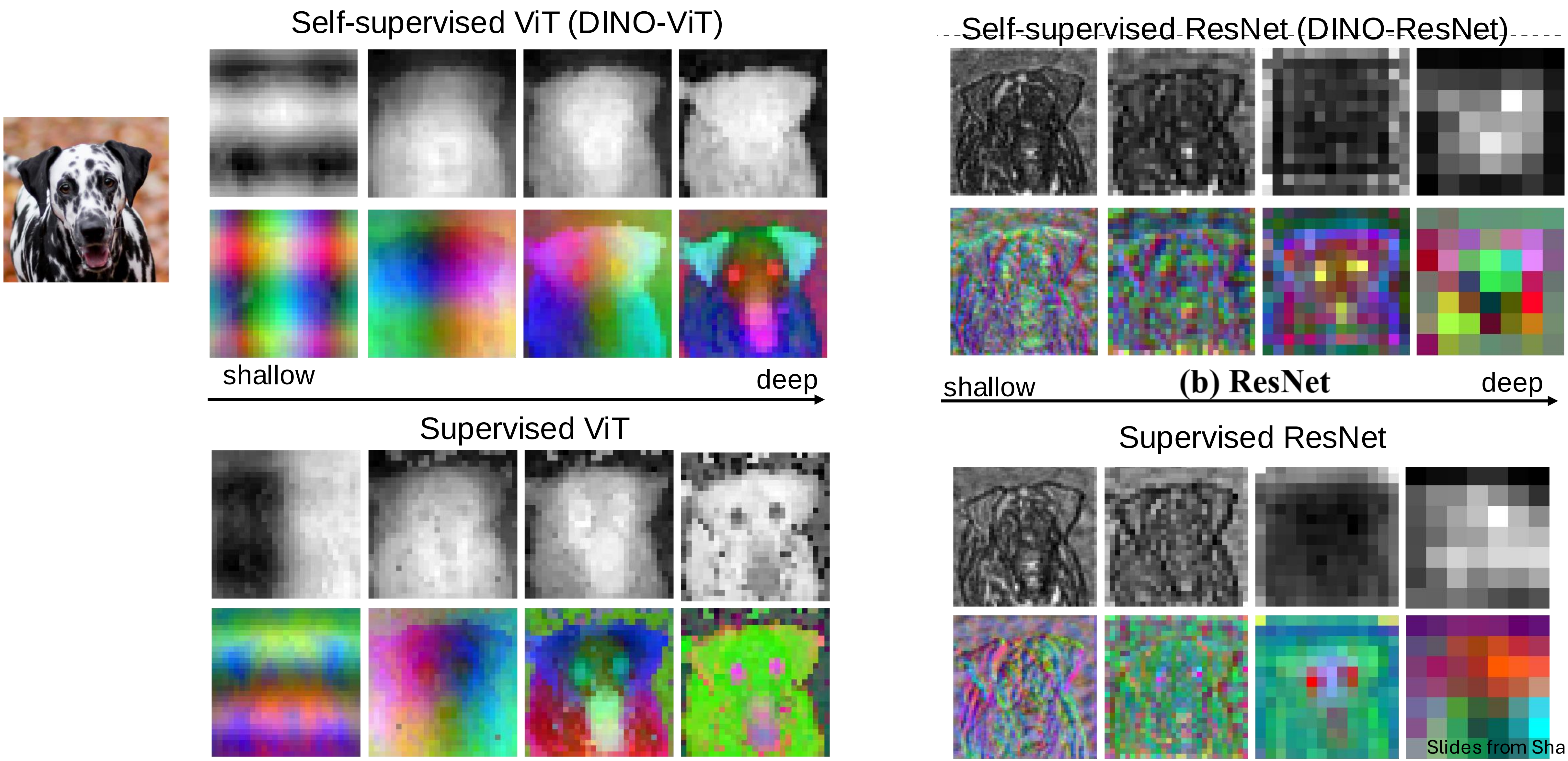
```
vit_model = torch.hub.load('facebookresearch/dino:main',
                           f'dino_vits16', pretrained=True)

img = imread('zebra.png')

attn_maps = vit_model.get_last_selfattention(img)

# Choose head, Get attention map of class token
attn_map = attn_maps[0, HEAD, 0, 1:].reshape((1, 1, H_PATCHES, W_PATCHES))
attn_map = F.interpolate(attn_map, scale_factor=16, mode="nearest")
```





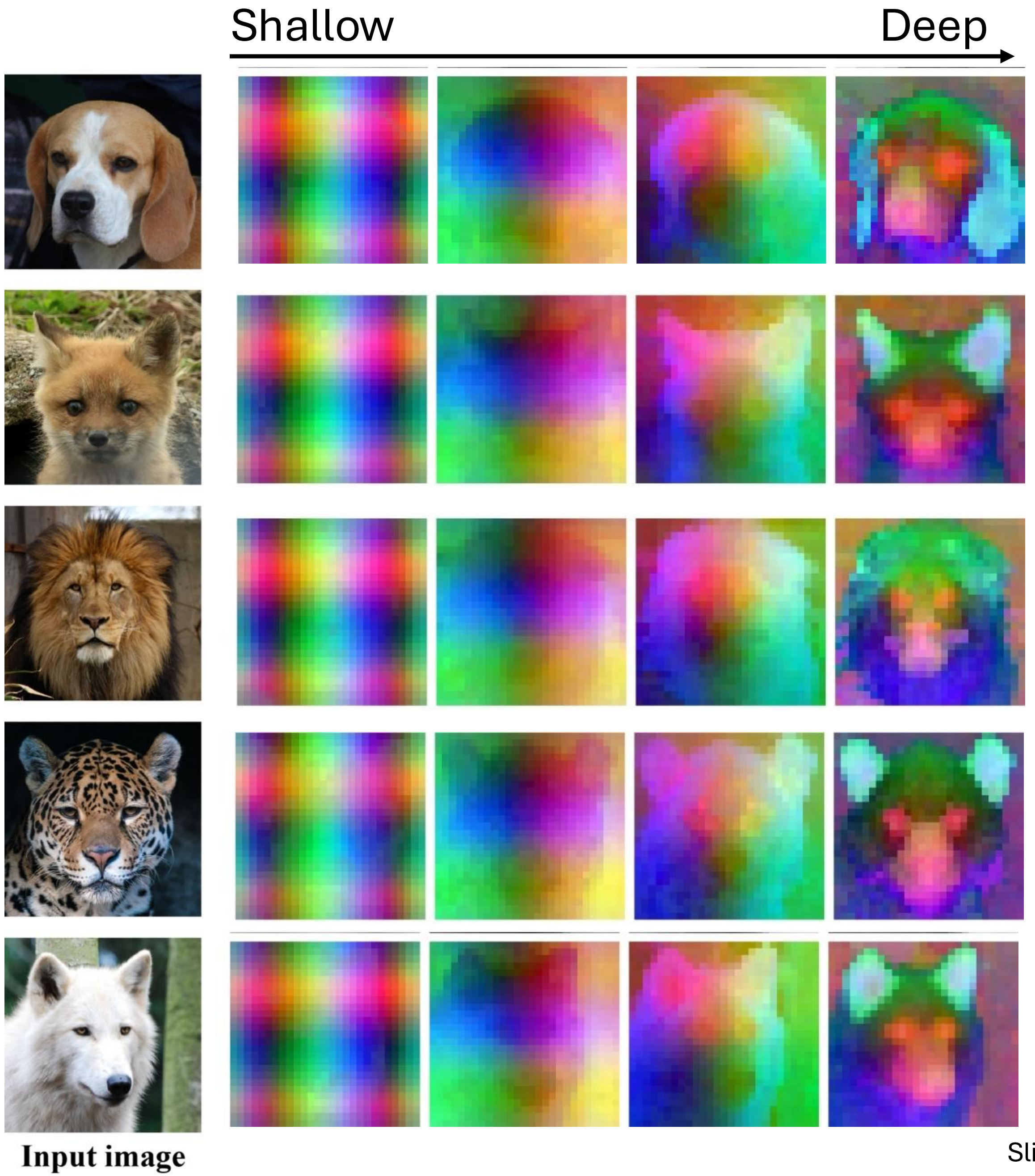
Slides from Shai Bagon

PCA (Keys) across layers

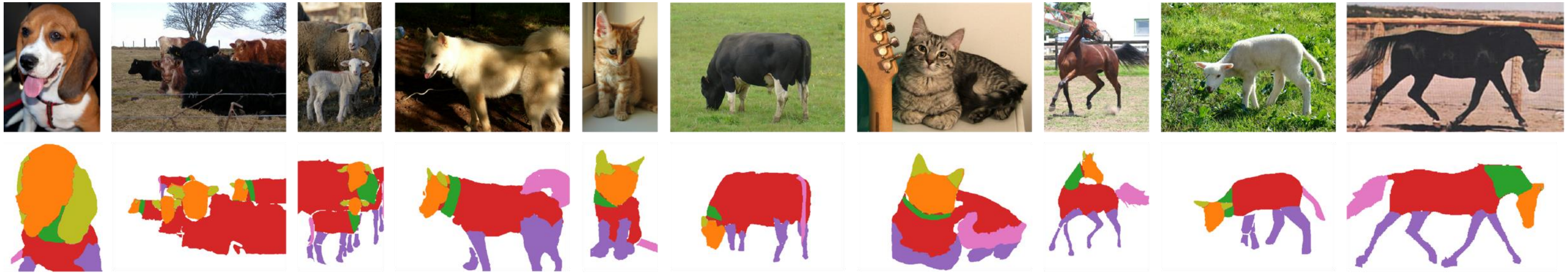


Slides from Shai Bagon

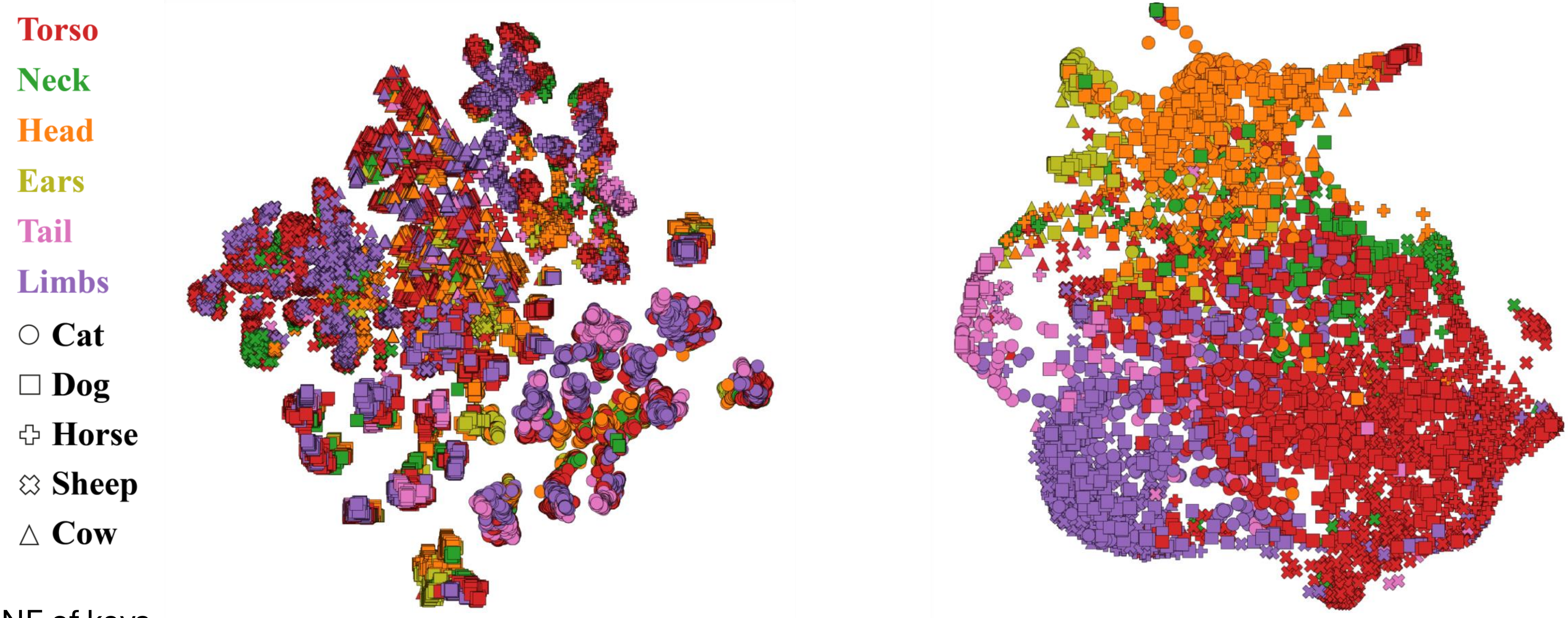
PCA – DiNO ViT



Amir, S., Gandelsman, Y., Bagon, S., Dekel, T.,
“Deep ViT Features as dense visual descriptors”



(a) Sample images and ground truth parts



(c) Supervised ViT

(b) Self-Supervised ViT (DINO-ViT)

T-SNE of keys visualization

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[CLS] as a **global appearance** representation

Tumanyan, N., Bar-Tal, O., Bagon, S., Dekel, T.
Splicing vit features for semantic appearance transfer (CVPR 2022)

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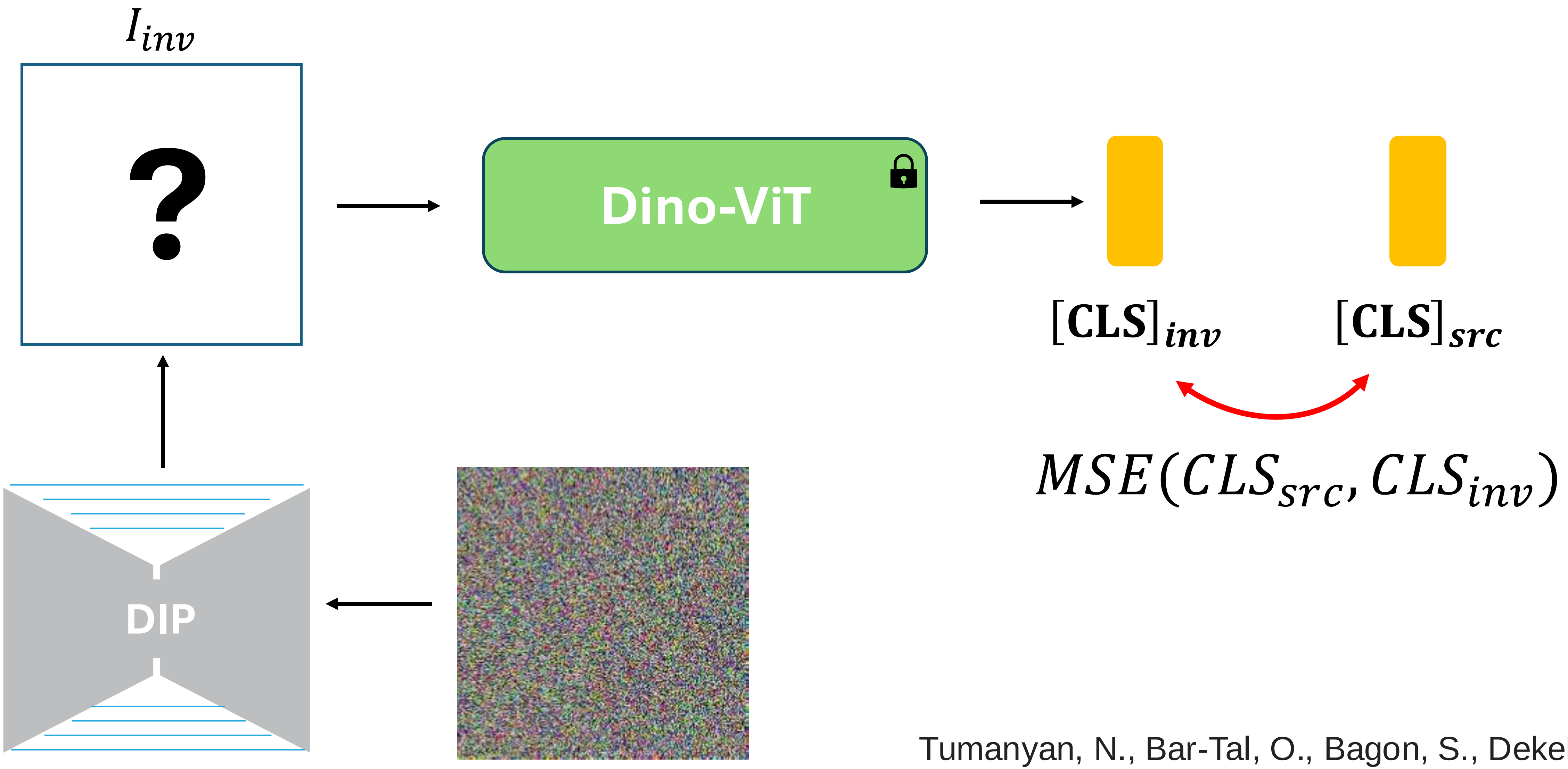
[CLS] as a **global appearance** representation



Tumanyan, N., Bar-Tal, O., Bagon, S., Dekel, T.
Splicing vit features for semantic appearance transfer (CVPR 2022)

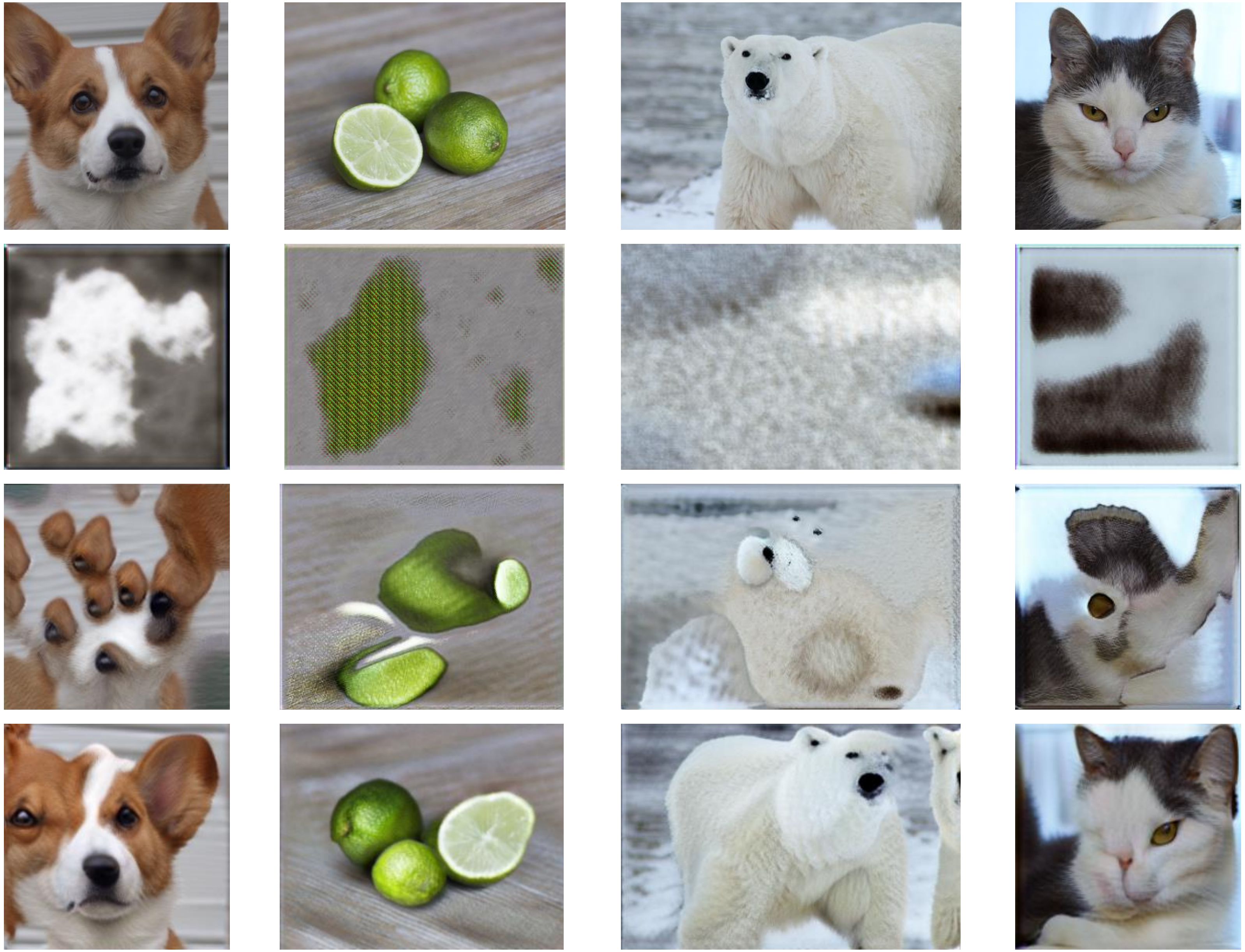
[CLS] as a **global appearance** representation

Input

layer **0**

layer **3**

layer **11**



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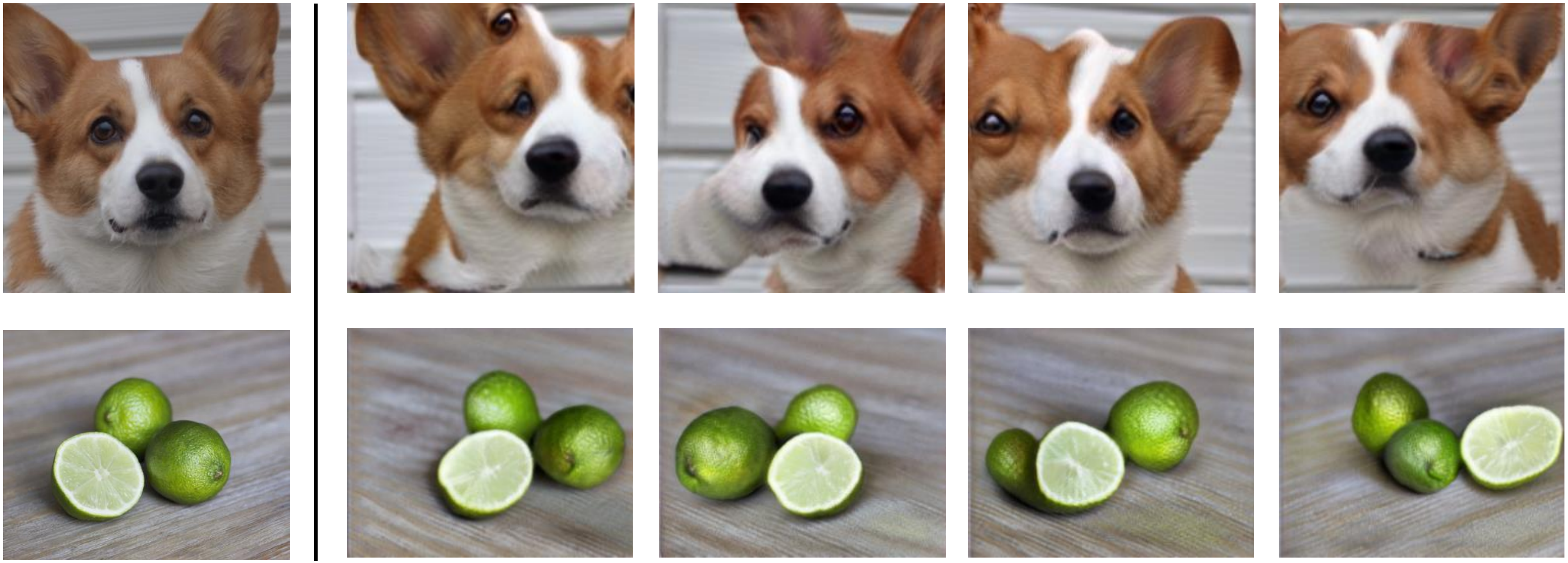
[CLS] as a **global appearance** representation



Inversion run 1 Inversion run 2 Inversion run 3 Inversion run 4

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