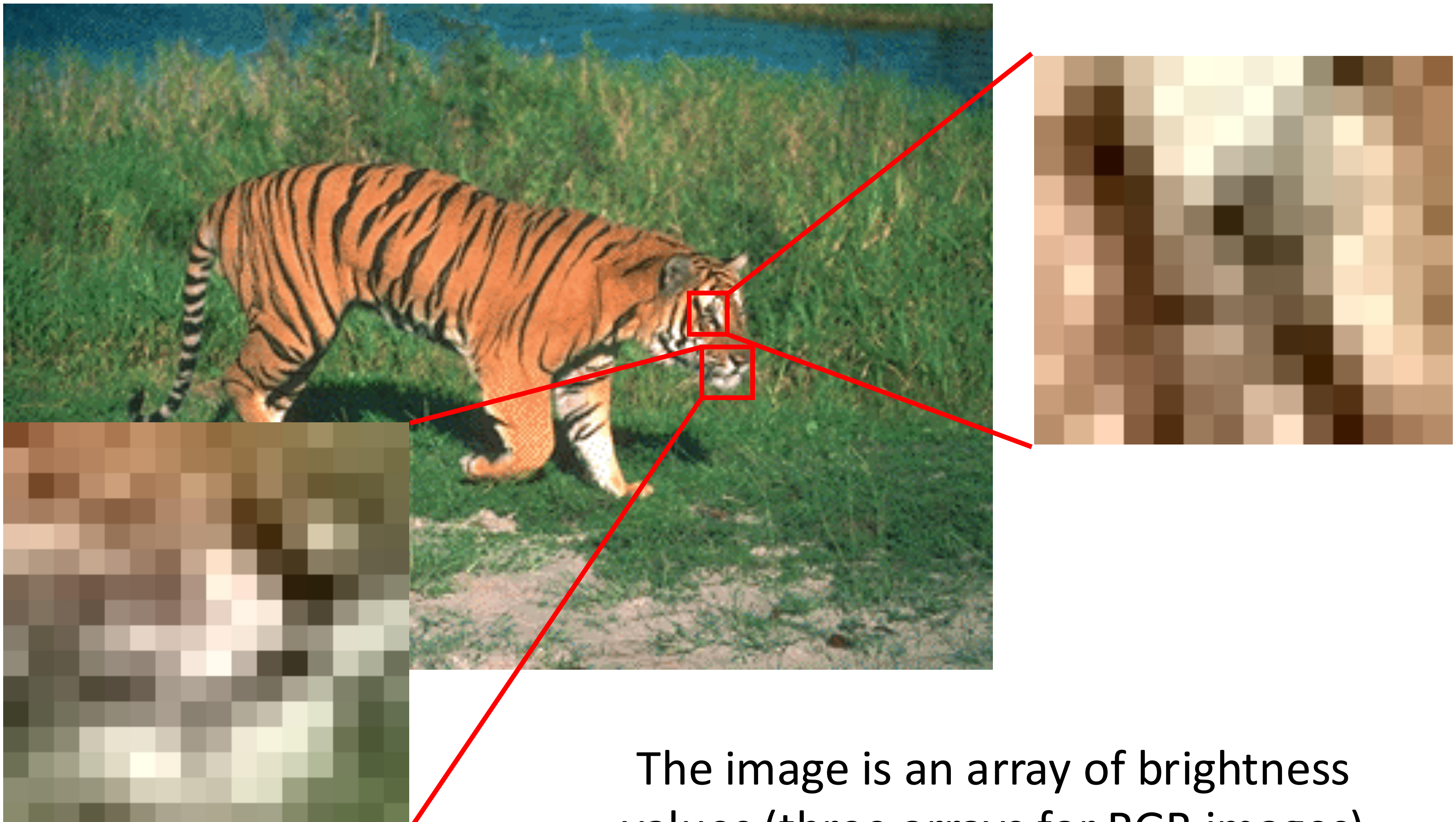
# Radiometry of Image Formation

Jitendra Malik

# What is in an image?



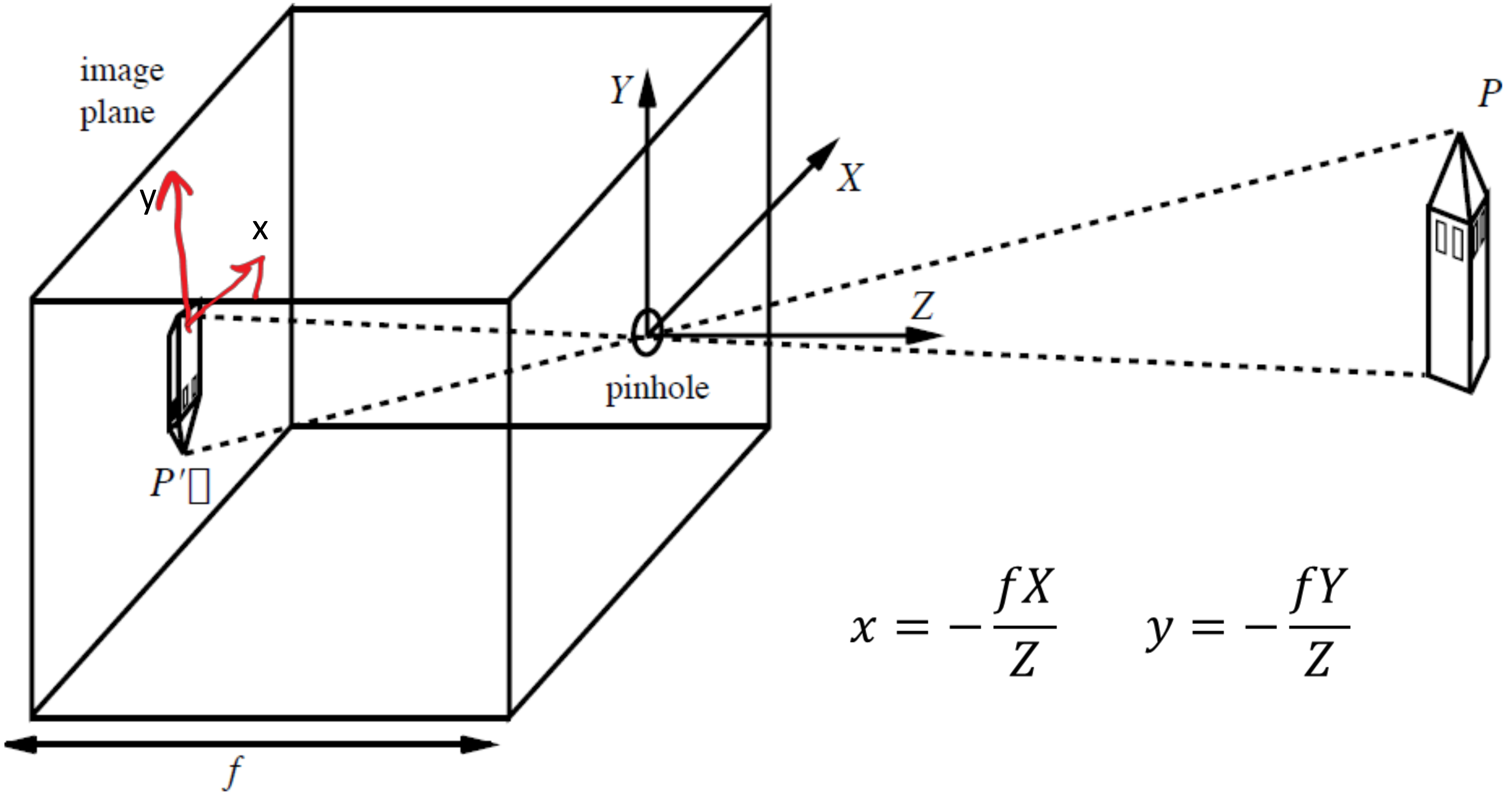
The image is an array of brightness values (three arrays for RGB images)

# A camera creates an image …


The image I(x,y) measures how much light is captured at pixel (x,y)

We want to know
- Where does a point (X,Y,Z) in the world get imaged?
- What is the brightness at the resulting point (x,y)?

# The pinhole camera models where a scene point is projected




$$x = -\frac{fX}{Z} \qquad y = -\frac{fY}{Z}$$

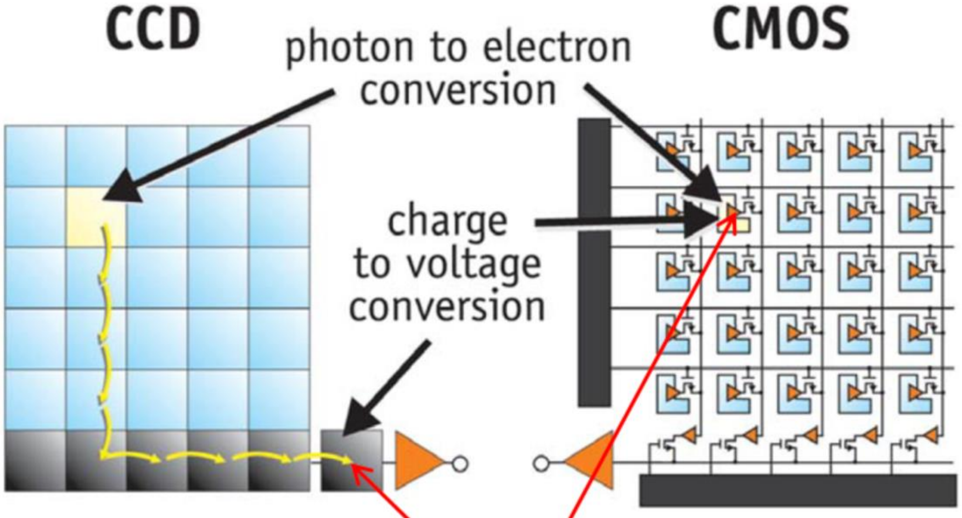
# Now let us try to understand brightness at a pixel (x,y) …


The image I(x,y) measures how much light is captured at pixel (x,y). Proportional to the number of photons captured at the sensor element (CCD/CMOS/rod/cone/..) in a time interval.

We use the scientific term IRRADIANCE for this concept. Irradiance is defined as the radiant power per unit area, and has units W/$m^2$. Usually denoted by E.

# Digital Cameras

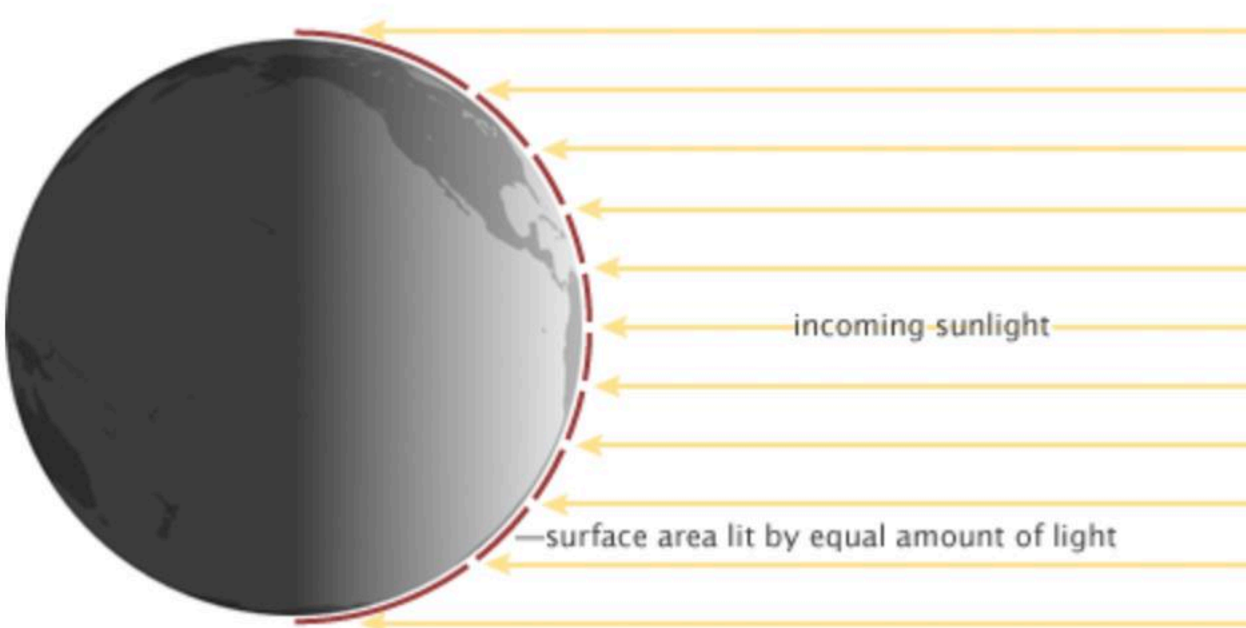
- The Charge Coupled Device was invented in 1969, based on exploiting the fact that silicon atoms can release electrons when hit by photons. Nowadays CMOS imagers are more common. The key difference is in how the charge generated is converted to a voltage (when digitized this becomes the pixel brightness value)



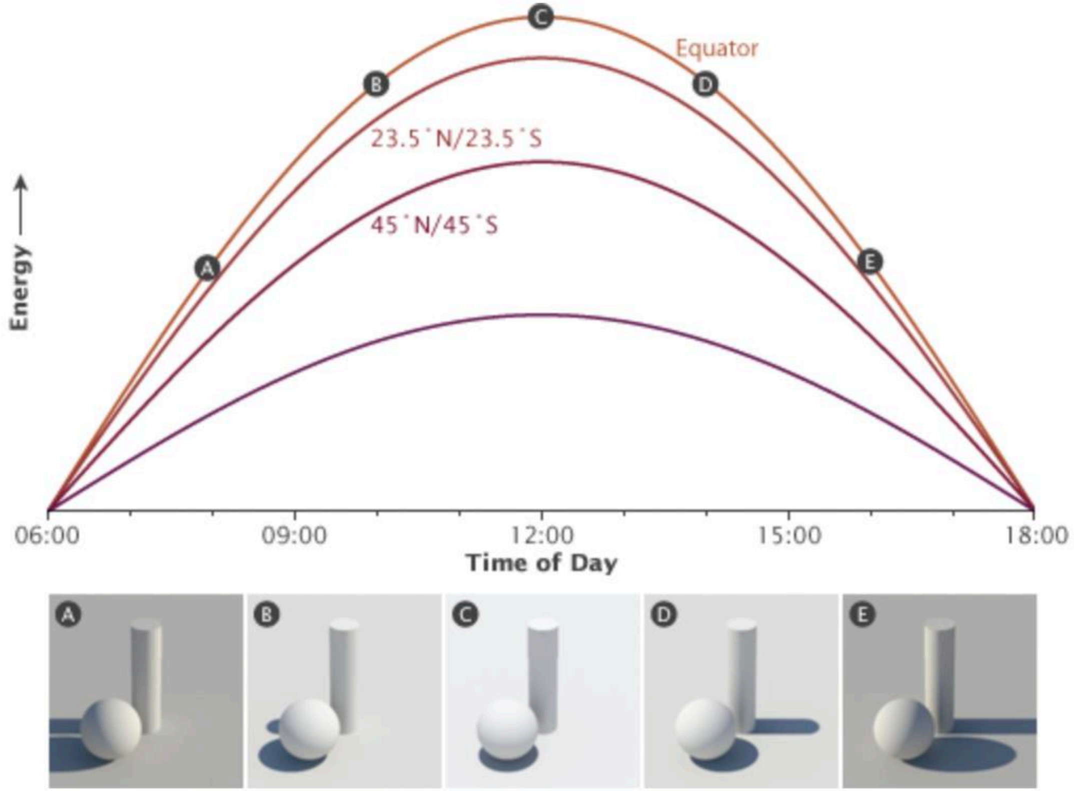

*CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.*

The response at a pixel is proportional to $E \, \Delta A \, \Delta t$

# Irradiance on earth due to the sun



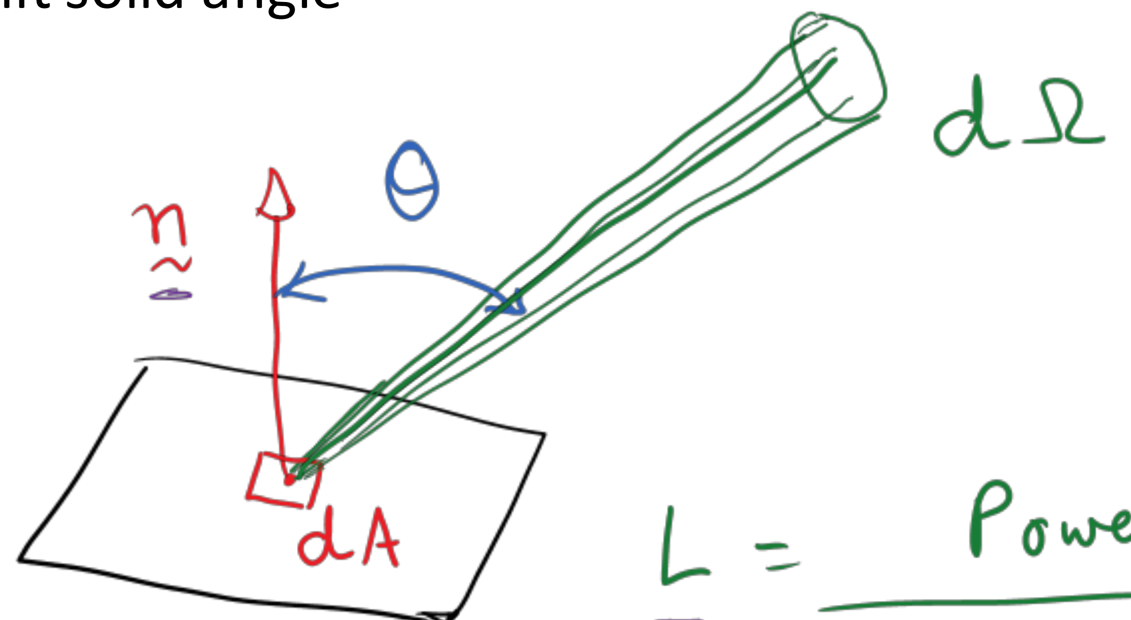

At Earth's average distance from the Sun (about 150 million kilometers), the average irradiance due to solar energy reaching the top of the atmosphere directly facing the Sun is about 1,360 watts per square meter




The solar radiation received at Earth's surface varies by time and latitude. This graph illustrates the relationship between latitude, time, and solar energy during the equinoxes. The illustrations show how the time of day (A-E) affects the angle of incoming sunlight (revealed by the length of the shadow)

# Radiance is a directional quantity

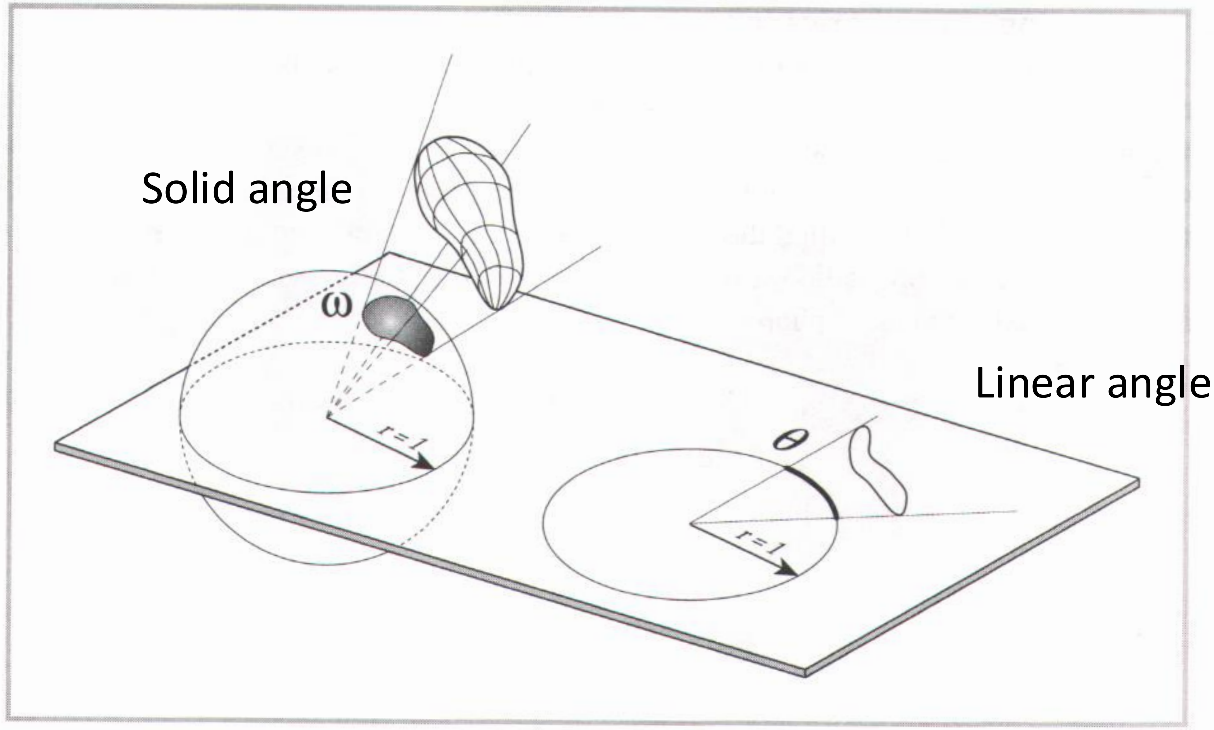
Radiant power travelling in a given direction per unit area (measured perpendicular to the direction of travel) per unit solid angle




$$L = \frac{\text{Power}}{(dA \cos\theta)(d\Omega)}$$

units are $W\, m^{-2}\, sr^{-1}$

Read more on Wikipedia




viewing direction must be the same direction as the normal to the arc or the patch, otherwise we need to correct by a factor of $\cos\alpha$ where $\alpha$ is the angle between the viewing direction and the normal. Thus, the foreshortened angle in the 2D plane is

$$\theta = \frac{s}{r}\cos\alpha$$

and the foreshortened angle for an area in 3D space is

$$\omega = \frac{A}{r^2}\cos\alpha$$

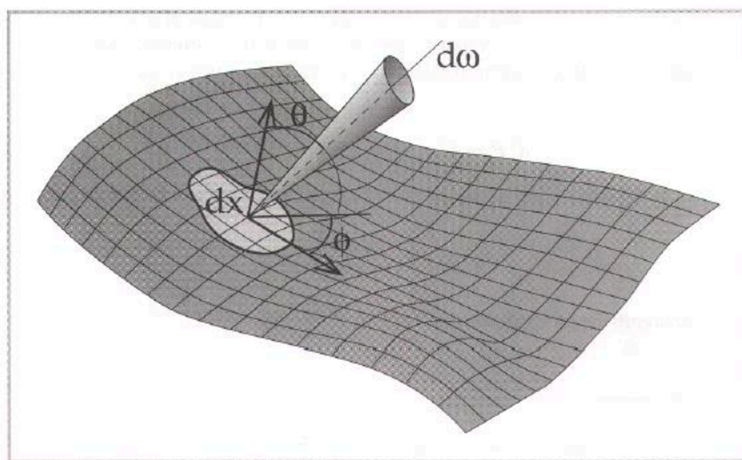
# Outgoing radiant power

The power per unit area leaving a point is obtained by integrating over the solid angle

$$\int_{\omega} L \cos\theta d\omega$$

where

$$d\omega = \sin\theta d\theta d\phi$$

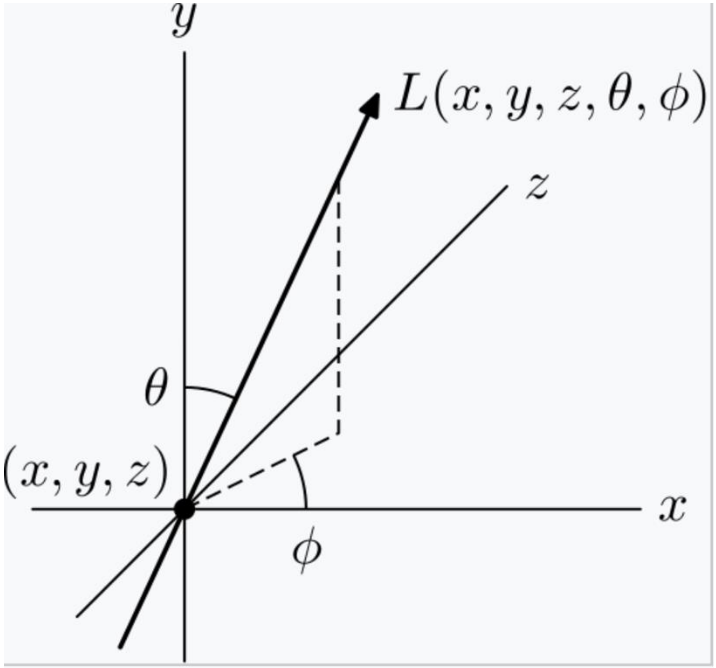


# Light Field
## (also called plenoptic function)

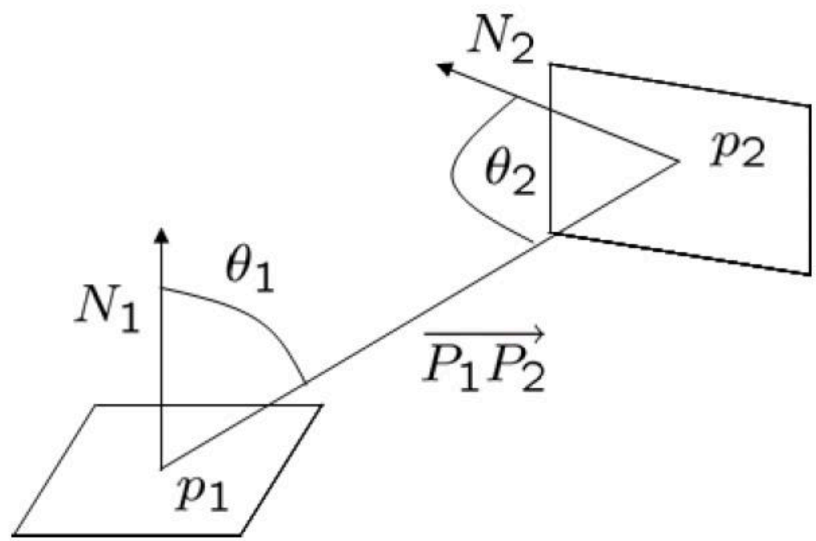
- Vector function describing amount of light (radiance) flowing in every direction through every point in space. It is a 5D function.

It turns out that in vacuum, radiance is conserved along a ray, so it is just 4D

Let's prove this!



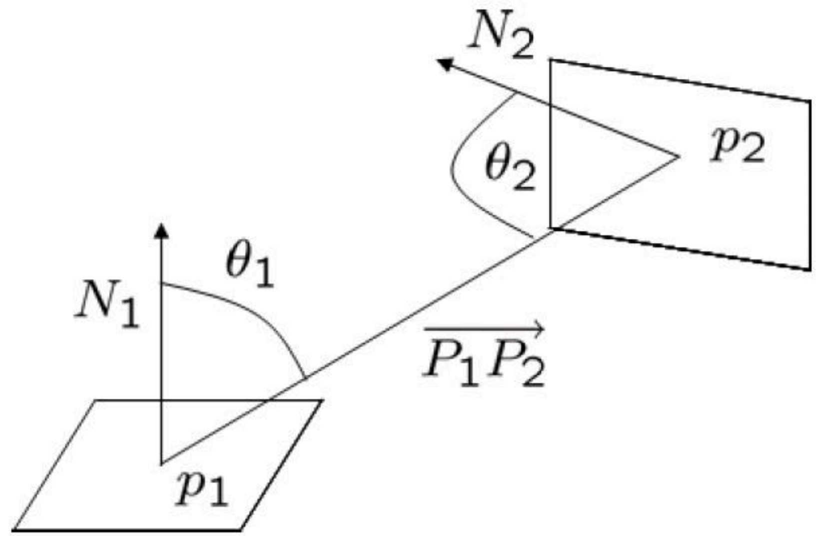

Parameterizing a ray in 3D space by position ($x$, $y$, $z$) and direction ($\theta$, $\phi$).




The power leaving from $P_1$ to $P_2$ per solid angle is

$$L(P_1, \overrightarrow{P_1P_2})dA_1 \cos\theta_1$$

where the $dA_1 \cos\theta_1$ is the foreshortened area of $P_1$ as seen by $P_2$.



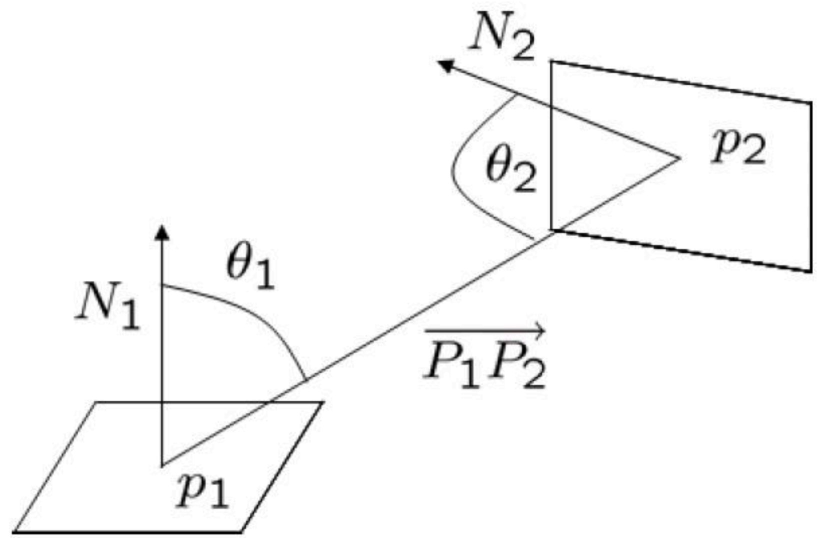

The power leaving from $P_1$ to $P_2$ per solid angle is

$$L(P_1, \overrightarrow{P_1P_2})dA_1 \cos\theta_1$$

where the $dA_1 \cos\theta_1$ is the foreshortened area of $P_1$ as seen by $P_2$.

The solid angle of $P_2$ as seen by $P_1$ is

$$\frac{dA_2 \cos\theta_2}{r^2}$$




The power leaving from $P_1$ to $P_2$ per solid angle is

$$L(P_1, \overrightarrow{P_1P_2})dA_1 \cos\theta_1$$

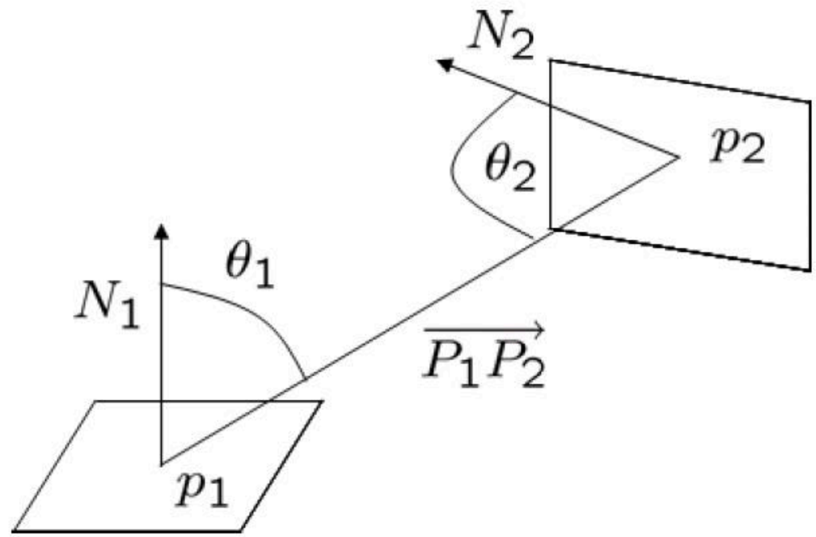
where the $dA_1 \cos\theta_1$ is the foreshortened area of $P_1$ as seen by $P_2$.

The solid angle of $P_2$ as seen by $P_1$ is

$$\frac{dA_2 \cos\theta_2}{r^2}$$

So putting this together, we get for the power leaving $P_1$

$$L(P_1, \overrightarrow{P_1P_2})dA_1 \cos\theta_1 \frac{dA_2 cos\theta_2}{r^2}$$




Similarly from $P_2$'s perspective, we get the power reaching $P_2$

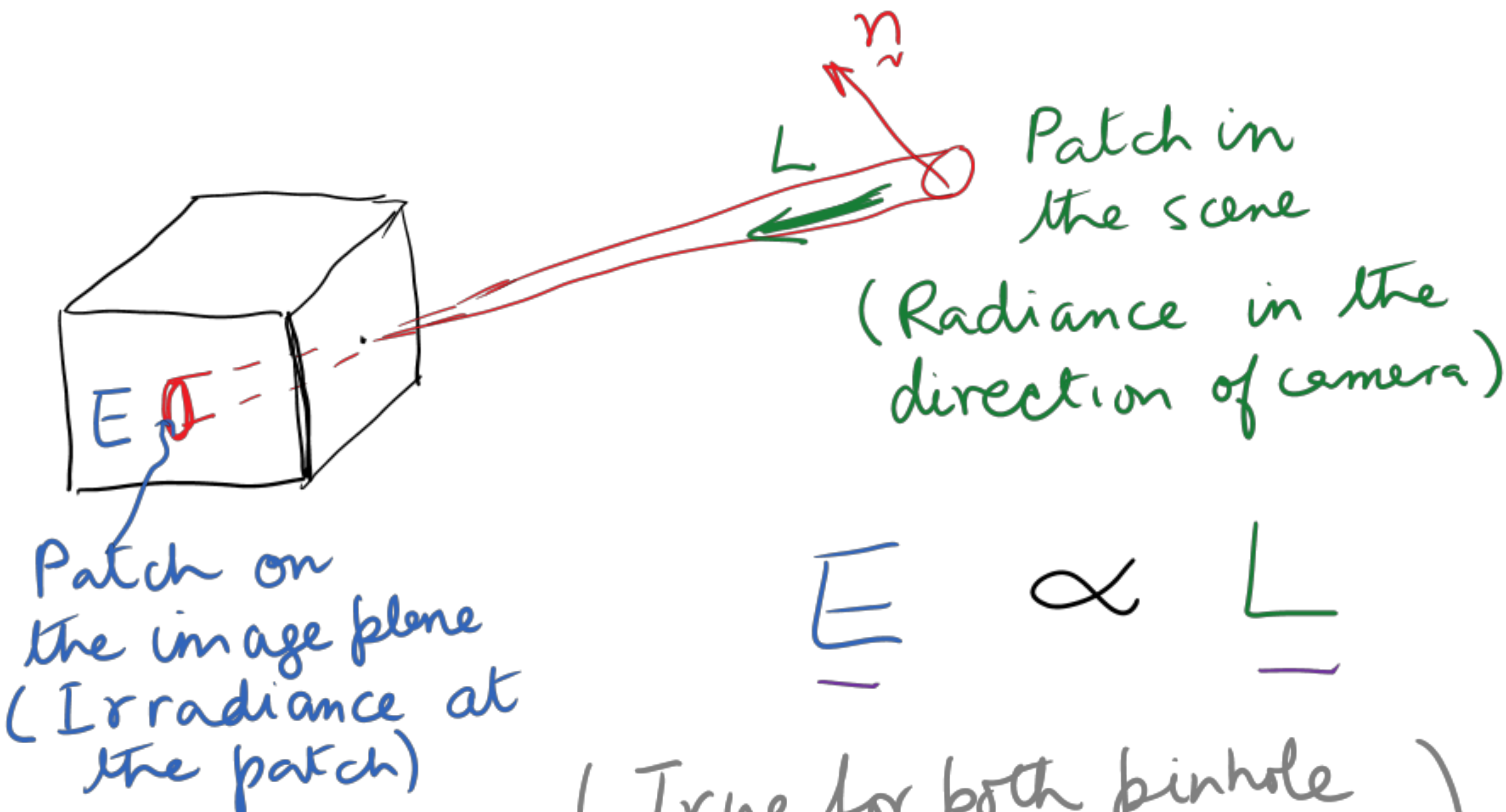
$$L(P_2, \overrightarrow{P_1P_2}) dA_2 cos\theta_2 \frac{dA_1 \cos\theta_1}{r^2}$$

As they are equal, the radiance terms must be equal, and we have shown that radiance along the ray is conserved.

# Quiz question

- What about the inverse square law? Light intensity is supposed to fall off with distance squared.

# Image irradiance is proportional to scene radiance in the direction of the camera




$$E \propto L$$

(True for both pinhole cameras & lens systems)

# For a camera with a lens

While we conventionally use a pinhole model, we now examine the effect of imaging with a lens. It can be shown that (see Forsyth and Ponce)

$$E = \left[\pi \left(\frac{d}{f}\right)^2 \cos^4 \alpha\right] L$$

In the equation above, $E$ is the irradiance at the sensor on a camera, and this is what is measured at each pixel in a CCD camera. $L$ is the radiance from the scene at a patch in the direction of the camera. Sensitive eyes (such as of nocturnal animals) are usually bigger as increasing $d$ increases the amount of light that enters the eye. The focal length $f$ is not usually reduced, as this means a larger viewing angle for each photoreceptor, leading to reduced spatial resolution of the eye.

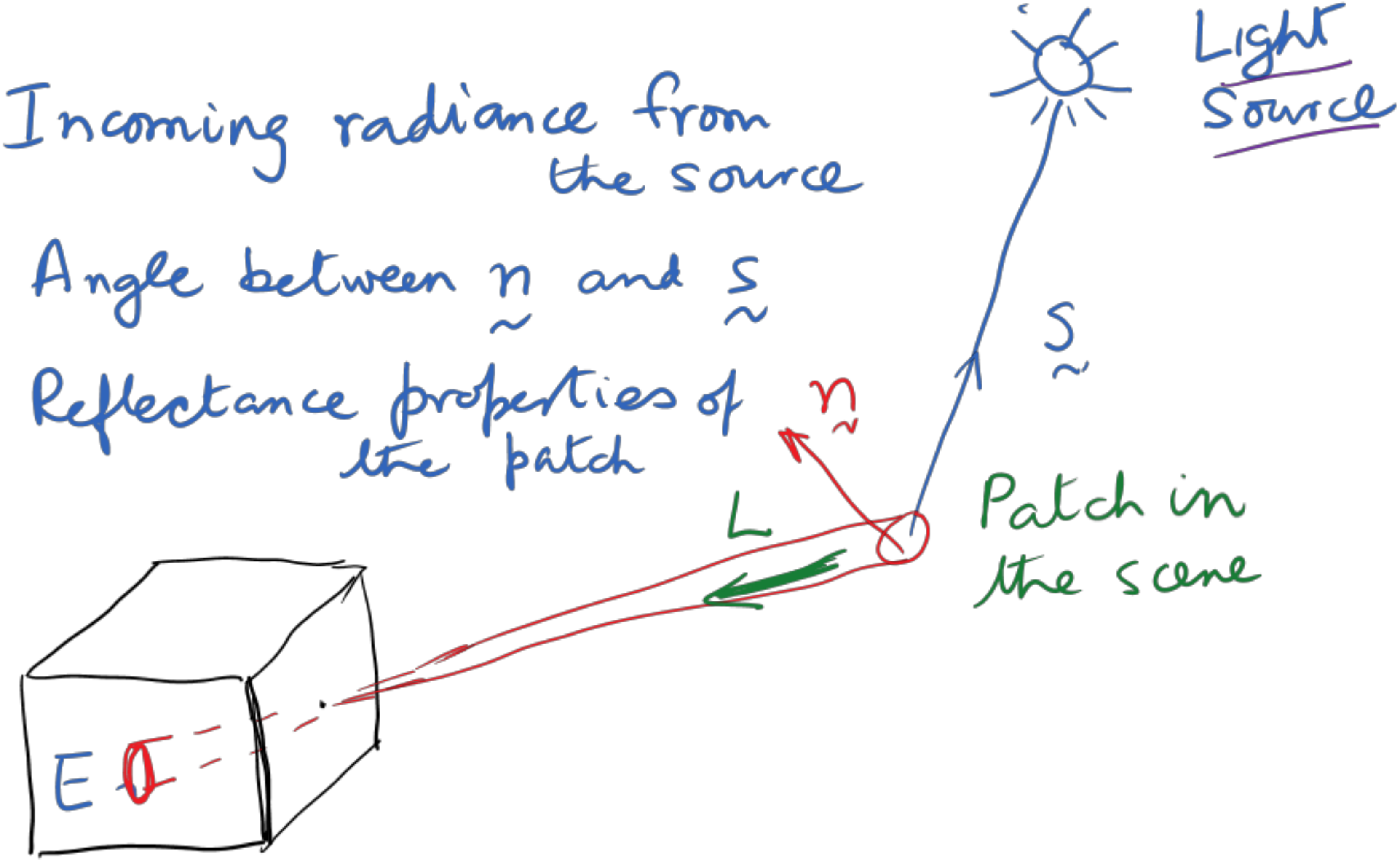
Thus a camera measures radiance in the direction of the camera.

# Capturing light fields

- If you want to measure it around an object, just take lots of photographs from multiple viewpoints surrounding the object. Basis of NeRF (Neural Radiance Field) reconstructions.
- Or you can buy a “light-field camera” (Lytro makes one). These use micro-lens arrays to capture rays from multiple directions.

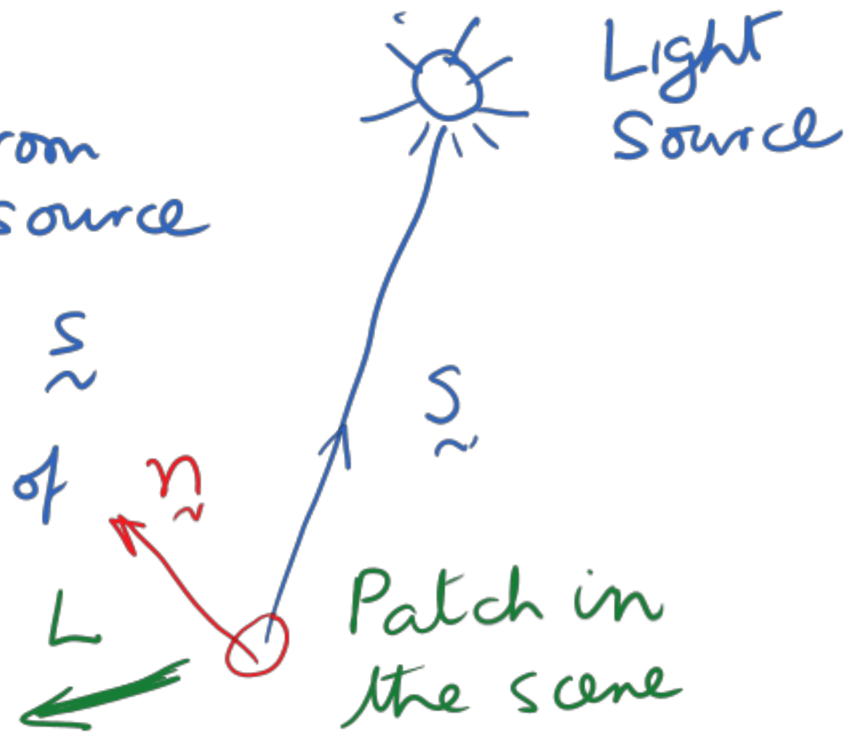
# What causes the outgoing radiance at a scene patch?

- Incoming radiance from the source
- Angle between $\underset{\sim}{n}$ and $\underset{\sim}{s}$
- Reflectance properties of the patch



# What causes the outgoing radiance at a scene patch?

- Incoming radiance from the source
- Angle between $\underset{\sim}{n}$ and $\underset{\sim}{s}$
- Reflectance properties of the surface patch




Two special cases:

- Specular surfaces - Outgoing radiance direction obeys angle of incidence=angle of reflection, and co-planarity of incident & reflected rays & the surface normal.
- Lambertian surfaces - Outgoing radiance same in all directions

# The Lambertian model

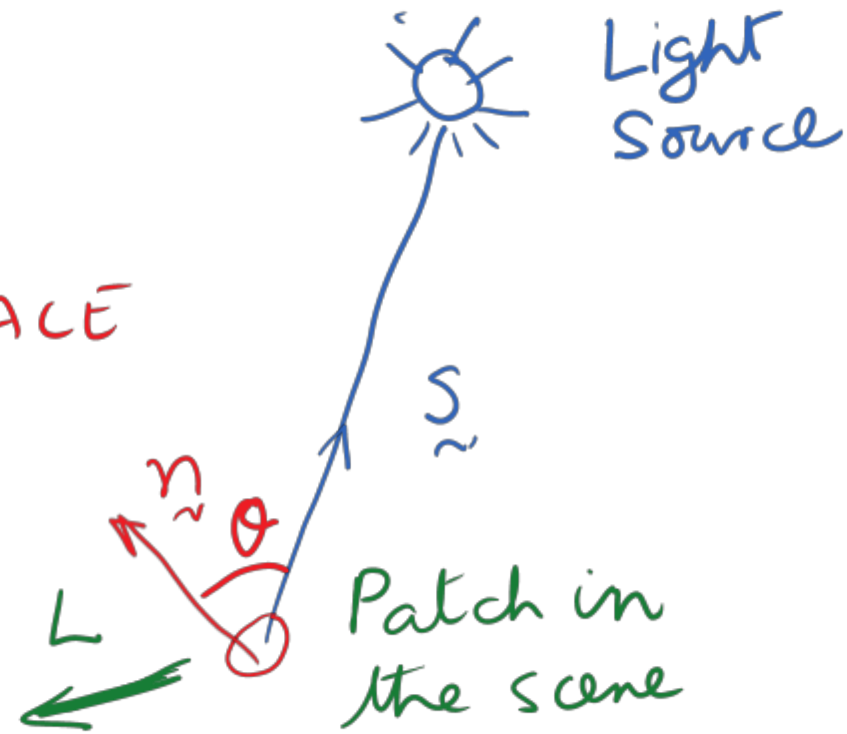
$$L = \rho \lambda \underset{\sim}{n} \cdot \underset{\sim}{s}$$

$\rho$ = ALBEDO OF SURFACE

0 ⟺ BLACK

1 ⟺ WHITE

$\lambda$ depends on radiance of light source




$$\underset{\sim}{n} \cdot \underset{\sim}{s} = \cos\theta$$ corresponds to foreshortening

We often model reflectance by a combination of a Lambertian term and a specular term. If we want to be precise, we use a BRDF (Bidirectional Reflectance Distribution function) which is a 4D function corresponding to the ratio of outgoing radiance in a particular direction to the incoming irradiance in some other direction. This can be measured empirically.

# Bidirectional Reflectance Distribution Function

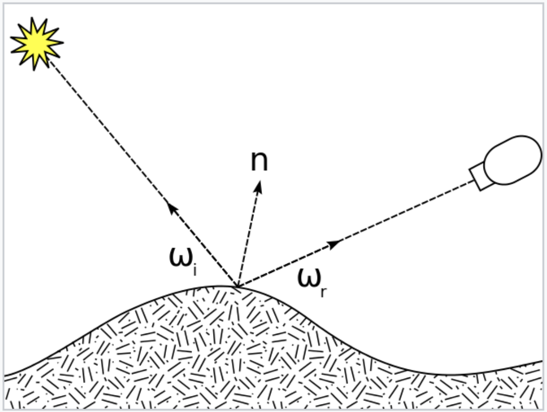



Diagram showing vectors used to define the BRDF. All vectors are unit length. $\omega_{\mathrm{i}}$ points toward the light source. $\omega_{\mathrm{r}}$ points toward the viewer (camera). $n$ is the surface normal.

$$f_{\mathrm{r}}(\omega_{\mathrm{i}},\,\omega_{\mathrm{r}}) \,=\, \frac{\mathrm{d}\,L_{\mathrm{r}}(\omega_{\mathrm{r}})}{\mathrm{d}\,E_{\mathrm{i}}(\omega_{\mathrm{i}})} \,=\, \frac{\mathrm{d}\,L_{\mathrm{r}}(\omega_{\mathrm{r}})}{L_{\mathrm{i}}(\omega_{\mathrm{i}})\cos\theta_{\mathrm{i}}\,\,\mathrm{d}\,\omega_{\mathrm{i}}}$$

where $L$ is radiance, or power per unit solid-angle-in-the-direction-of-a-ray per unit projected-area-perpendicular-to-the-ray, $E$ is irradiance, or power per unit *surface area*, and $\theta_{\mathrm{i}}$ is the angle between $\omega_{\mathrm{i}}$ and the surface normal, $\mathbf{n}$. The index $\mathrm{i}$ indicates incident light, whereas the index $\mathrm{r}$ indicates reflected light.

Given the BRDF, and the incoming radiance, we can predict the outgoing radiance in all directions

# Real world scenes have additional complexity…

- Objects are illuminated not just by light sources, but also by reflected light from other surfaces. In computer graphics, ray tracing and radiosity are techniques that address this issue.
- Shadows